

# THE RECONSTRUCTION OF SHARPE

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# Chapter 1

## Introduction

### 1.1 Symbolic Hierarchical Automated Reliability and Performance Evaluator(SHARPE)

Today's computer system design has become more and more complicated, so it is hard to predict the reliability, availability and serviceability characteristics of the resulting system. Also, it is too expensive and time-consuming to build even one prototype to take measurements. Even when that is not the case, if the model is a good match for the system, designers can more easily and quickly carry out trade-off studies, and compare design alternatives.

Generally, there are two kinds of models, discrete-event simulation models and analytic models, to help designers predict system behavior without having to build and measure a real system. For discrete-event simulation models, designers build a program to reproduce the running behavior of the modeled system and take measures of the behavior. On the other hand, for analytic models, designers use a set of formulas or equations to describe the system. By solving these equations, designers get the measures of the system. Although discrete-event simulation models provide more details of the system behavior, they consume more time and more computer resources than analytic models. The situation may become worse when designers want to vary many of the parameters of the system for many times. Analytic models are better abstractions of systems. But analysts have to be very careful on how to abstract these real-world systems.

SHARPE (Symbolic Hierarchical Automated Reliability and Performance Evaluator)

is a software tool that analyzes a specific class of analytic models – stochastic models. It accepts a specification language, called SHARPE language, for building single or hierarchical combinations of analytic models and for choosing proper algorithms for analyzing them. Originally, SHARPE provided analysis algorithms for the following model types:

- Reliability block diagrams
- Fault trees
- Reliability graphs
- Series-parallel acyclic directed graphs
- Single-chain and multiple-chain product-form queueing networks
- Markov and semi-Markov chains
- Generalize Stochastic Petri nets

SHARPE language gives users the power to choose models that are a proper match of the problem under investigation and it is up to users to interpret the parameters of the system and the results of measurements in a meaningful way. So, users can freely deploy all the above models on any systems if necessary. In the SHARPE test-bed, different system examples, such as multiprocessor system, wireless system, software system, and token ring system, etc., are included. Another big plus for SHARPE is that it supports hierarchical modeling, which can solve very complicated systems without causing stiffness or largeness.

Programming of SHARPE began in the early 1980s, in C language. The first version appeared at 1986. At that time, computer world was still lacking the ideas of compiling tools such as lex and yacc. As time passed, more and more models have been added into

SHARPE which has gradually made the code, especially the language parsing part, difficult to manage. It has become more and more difficult to add new model types into SHARPE or to extend the SHARPE language syntax. So, **flex** – an advanced version of *lex*, and **Bison** – an advanced version of *yacc* have been used to reconstruct SHARPE. The C language compiler used is **GCC**. Introduction to **flex**, **Bison** and **GCC** is given at the section 1.2. Details of work that has been done in this project are listed at the section 1.3.

## 1.2 Tools used

### 1.2.1 GCC

**GCC** stands for "GNU Compiler Collection", where **GNU** was chosen following a hacker tradition, as a recursive acronym for "GNU's Not Unix". **GCC** can compile programs written in C, C++, Objective C, Fortran, Java and CHILL. The main goal of **GCC** was to provide a good, fast compiler for computer platforms in the class that the GNU system aims to run on: 32-bit machines with 8-bit addresses bytes and several general registers, include AIX, DOS, HP-UX, SCO OpenServer/Unixware, Solaris (SPARC, Intel), SGI, and Windows 95, 98, NT, 2000. So, having been compiled successfully by **GCC**, SHARPE can easily be deployed on those popular platforms.

### 1.2.2 flex

**flex**, also from GNU, is a tool for generating *lexical scanners*, which are programs for recognizing lexical patterns in text. At first, **flex** reads a description of a lexical scanner from the given input files, or its standard input if no file names are given. The description is in the form of pairs of regular expressions and C code, called *rules*. According to the description, **flex** generates a C source file, '**lex.yy.c**', which defines a routine '**yylex()**'. This



file should be compiled and linked with the **'-lfl'** library to produce an executable. When the executable is running, it analyzes its input for occurrences of the regular expressions. Whenever it finds one, it executes the corresponding C code.

The **flex** input file consists of four sections, separated by a line with just **'%%'** in it:

**%{**

*C declarations*

**%}**

*definitions*

**%%**

*rules*

**%%**

*Additional C code*

The *C declarations* section may define types and variables used in the actions. One can also use preprocessor commands to define macros, and use **#include** to include header files that do any of these things.

The *definitions* section contains declarations of simple *name* definitions to simplify the scanner specification, and declarations of *start* conditions, which supports conditionally activating rules.

The *rules* section of the **flex** input contains a series of rules of the form:

**pattern** *action*

where the pattern must be un-indented, which is written using an extended set of regular

expressions, and the action must begin on the same line, which can be any arbitrary C statement.

The *additional C code* section can contain any C code one wants to use.

The reason to choose **flex** rather than **lex** is that **lex** cannot handle languages, such as SHARPE language, having too many tokens.

### 1.2.3 **Bison**

**Bison**, as a GNU tool, is a general-purpose parser generator that converts a grammar description for an **LALR** context-free grammar into a C program to parse that grammar. It is upward compatible with **Yacc**: all properly-written **Yacc** grammars ought to work with **Bison** without change. **Bison** reads a **Bison** grammar file as input. The output is a C source file defining a function named **yyparse**, and the file is called a **Bison** parser. The job of the **Bison** parser is to group tokens into sets according to the grammar rules – for example, to group identifiers and operations into expressions. when it does this, it runs the actions for the grammar rules. The tokens come from a function called the *lexical scanner*, which, in this project, is the function **yylex** generated by **flex**.

The general form of a **Bison** grammar file is as follows:

```
%{
```

```
C declarations
```

```
%}
```

```
Bison declarations
```

```
%%
```

*Grammar rules*

%%

*Additional C code*

The *C declarations* may define types and variables used in the rules' actions. You can also use preprocessor commands to define macros used there, and use **#include** to include header files that do any of these things.

The *Bison declarations* declare the names of the terminal and non-terminal symbols, and may also describe operator precedence and the data types of semantic values of various symbols.

The *grammar rules* define how to construct each non-terminal symbol from its parts. The following rule defines a non-terminal *line* as newline character:

```
line : '\n'  
;
```

The *additional C code* can contain any C code one wants to use.

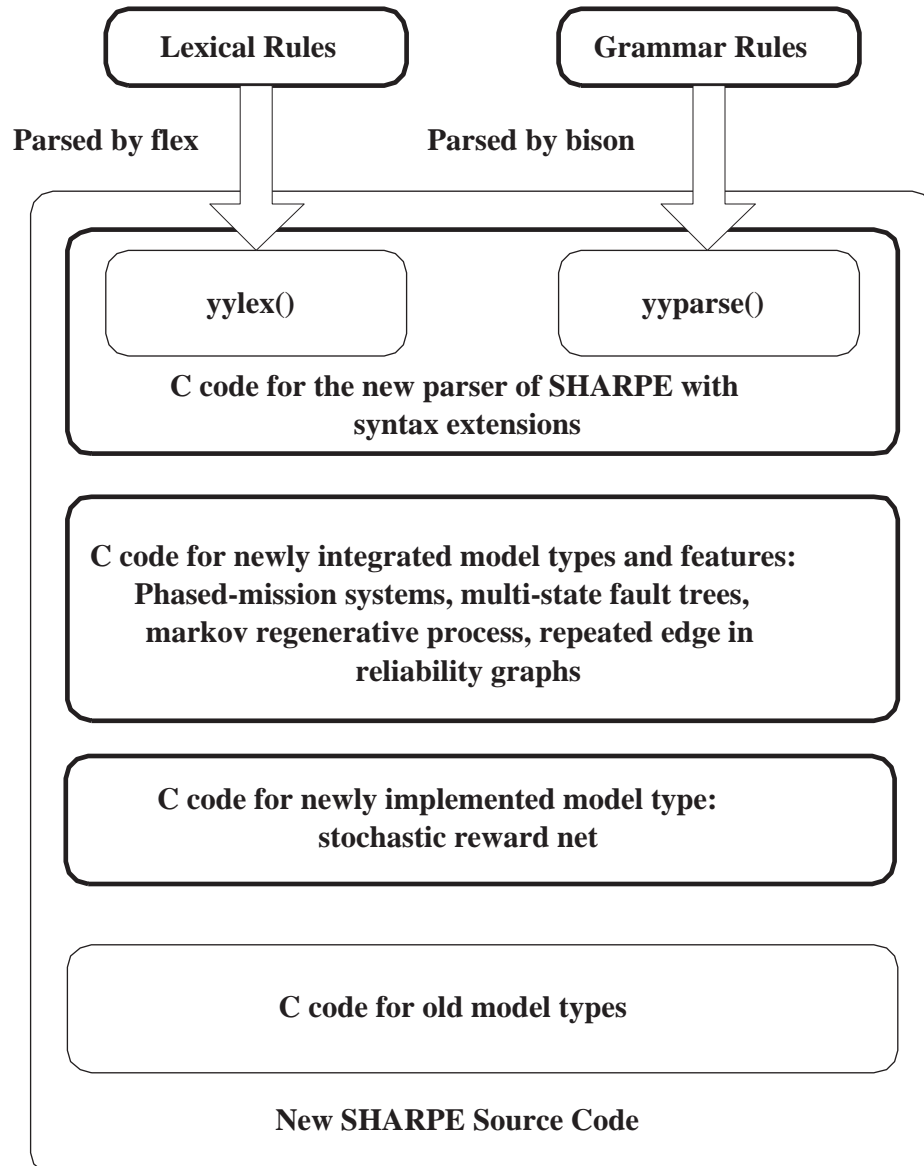
### 1.3 Work of reconstruction

The programming of SHARPE began in the early 1980s, in C language. First version was released at 1986. At that time, computer world was still lacking of the ideas of compiling tools such as lex and yacc. As time passed, more and more models have been added into SHARPE which has gradually made the code, especially the language parsing part, difficult to manage. It has become more and more difficult to add new model types into SHARPE. So, **flex** – an advanced version of lex, and **Bison** – an advanced version of yacc

have been used to reconstruct SHARPE (See Figure 1.1). Of course, the new version of SHARPE, which is backward compatible to the old version, supports old language syntax and all model types listed in section 1.1, Phased-mission systems, Multi-state fault trees, and repeated edges in reliability graphs from Xinyu Zang's work [18], Markov regenerative process from Wei Xie's work [17], and Stochastic Reward Nets, which is implemented by me. There is also fast Mean Time To Failure(MTTF) algorithm for Markov chains and semi-Markov chains [6], which is implemented by Wei Xie. All new changes to SHARPE are represented by rectangles with thick lines in Figure 1.1.

**The only exception** is the definition of a *name*. Now only any number of letters, digits, underline, and colon are used to define a *name*. *Names* can be any length, but SHARPE **only looks** at *the first 29 characters*, beyond that, SHARPE will ignore and provide a warning message to users.

**Another extension** to SHARPE language syntax is that *numbers* can be represented in scientific format so that 0.1 can be written as  $1.0E - 1$ , which can ease the burden on users when coding their SHARPE input files.



**Figure 1.1:** New SHARPE Construct

Other extensions to SHARPE language syntax will be mentioned when specific model types are introduced in subsequent chapters.

The new version of SHARPE accepts the following model types:

- Reliability block diagrams
- Fault trees
- Phased-mission systems
- Multi-state fault trees
- Reliability graphs with possibly repeated edges
- Series-parallel acyclic directed graphs
- Single-chain and multiple-chain product-form queueing networks
- Markov and semi-Markov chains
- Markov Regenerative Process
- Generalize Stochastic Petri nets
- Stochastic Reward Nets

There is also a test-bed which contains 41 directories and 978 test cases. The correctness of the new version of SHARPE is based on these test cases.

## **1.4 Scope of the thesis**

The remainder of this thesis is organized into 2 chapters, as follows. Chapter 2 introduces how Stochastic Reward Nets (SRNs) has been implemented in SHARPE. Chapter 3 intro-

duces all the model types which have been integrated into the new version of SHARPE. Examples have been selected to excise the features introduced. Appendix *A* includes all important data structures in SHARPE. Appendix *B* includes a partial SHARPE GUI document. Appendix *C* includes extra examples referenced in this thesis.

## Chapter 2

# New Model Type in SHARPE – Stochastic Reward Nets (SRNs)

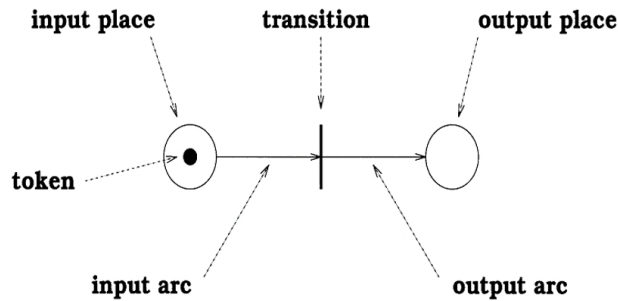
## 2.1 Background

### 2.1.1 Petri Nets (PNs) and Generalized Stochastic Petri Nets (GSPNs)

Petri nets (PNs) were introduced by C.A. Petri in 1962 [12]. As a bipartite directed graph, a PN consists two types of nodes: *places*,  $P$ , and *transitions*,  $T$ . Its directed arcs fall in two categories: *input arcs*, which lead from an *input place* to a transition, and *output arcs*, which connect a transition to an *output place*. Arcs cannot connect the same type of nodes, such as from places to places or from transitions to transitions. A non-negative number of *tokens* can be assigned to each place. A *marking*  $m \in \mathcal{M}$  is defined as a possible distribution of tokens to all places in the PN. Let  $P$  denotes the set of places. Then a marking  $m$  represents a multi-set,  $m \in \mathcal{M} \subset \mathcal{IN}^{|P|}$ , describing the number of tokens in each place. See Figure 2.1. We use circle to denote a place, and a rectangle or a bar to denote a transition. Places represent conditions in the system being modeled. Transitions represent events occurring in the system. *Input arcs* are directed arcs from places to transitions representing the requirement or conditions for the event, which is denoted by the transition, to be triggered; *output arcs* are directed arcs from transitions to places representing the state or condition resulting from the occurrence of an event; *input places* of a transition are the set of places that are connected to the transition through input arcs; *output places* of a transition are the set of places to which output arcs exist from the



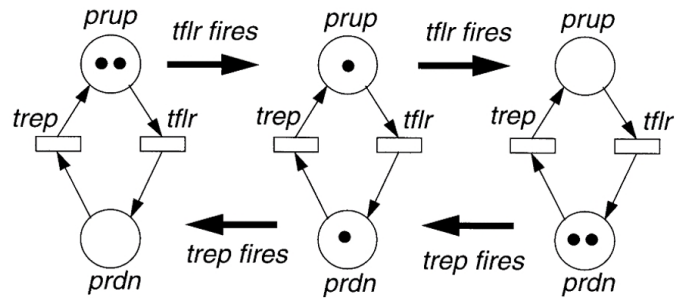
transition.



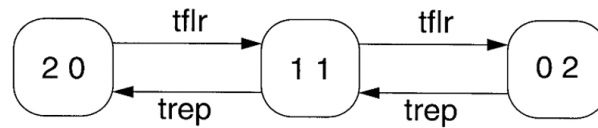
**Figure 2.1:** Basic components of a Petri net

A transition is *enabled* in the PN if the conditions for the corresponding event are met, which means all of the transition's input places contain at least one token. A transition is always enabled if there is no input arc connected to it. In the situation when more than one transitions is *enabled*, *priority* may be introduced to resolve the conflict (see Chapter 2.1.2). When an enabled transition *fires*, one token from each input place is removed and one token is added to each output place (See Figure 2.2). The firing of a transition may transform a PN from one marking into another, changing the state or condition. *Marking* of a Petri net is the distribution of tokens among the places of the net. Given an *initial* marking, the *reachability set*,  $\mathcal{RS}$ , is defined as the set of markings reachable through any firing sequences of transitions beginning from the initial marking (See Figure 2.2). A *reachability graph* is represented as a directed graph with markings as its nodes and marking-to-marking transitions as its directed arcs. Depending on the situation, a  $\mathcal{RS}$  could be infinite. Markings in which no transition is enabled are called *absorbing* markings.

Arcs of PNs can be extended to define *arc cardinality* or *multiplicity*. A transition is *enabled* when each input place connected to it contains at least as many tokens as the cardinality of the input arc. When the transition fires, the number of tokens removed from



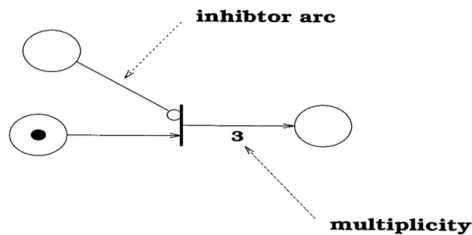
**Figure 2.2:** Enabling and Firing of Transitions



**Figure 2.3:** Reachability Set

the input place is the cardinality of the corresponding input arc, and the number of tokens added into the output place is the cardinality of the corresponding output arc (See Figure 2.4).

Further, *inhibitor arcs* are introduced as the third category of PN arcs. An inhibitor arc is drawn from a place to transition. The place is called *inhibitor place*. *Inhibitor arc* inhibits the firing of a transition when the corresponding inhibitor place has at least as many tokens as the cardinality of the corresponding inhibitor arc, even under the situation that all other conditions for enabling the transition are met. *Inhibitor arcs* are also directed arcs with a small circle rather than an arrow-head showing its direction (See Figure 2.4).



**Figure 2.4:** Extension of GSPN 1

Another way of extending PNs is to assign time with the firing of transitions, resulting in timed Petri nets. Generalized Stochastic Petri Nets (GSPNs) are one of them. In GSPNs, there are two types of transitions: *timed* transitions whose firing time is exponentially distributed and *immediate* transitions whose firing time is constant zero. *Timed* transitions are denoted by empty rectangles, while *immediate* transitions are drawn as bars.

The markings in the reachability set  $\mathcal{RS}$  of a GSPN are partitioned into two sets: the *vanishing* markings  $\mathcal{V}$  and the *tangible* marking  $\mathcal{T}$ . So,  $\mathcal{M} = \mathcal{V} \cup \mathcal{T}$ . Vanishing markings are those in which **at least one** immediate transition is enabled. Since vanishing markings are not resided in for any non-zero time and firings are acted instantaneously, the priority of immediate transitions is always higher than that of timed transitions.

Since computers have limited resource, only bounded GSPNs, whose underlying reachability sets are finite, are considered. Under the condition that only a positive number of transitions can fire in a finite time with non-zero probability, there is exactly one Continuous Time Markov Chain (CTMC) that corresponds to a given GSPN [10].

### 2.1.2 Stochastic Reward Nets (SRNs)

Stochastic Reward Nets (SRNs) are based on GSPN but extend them further [3]. Some of the most prominent extensions are revisited in the following: priorities, guards, marking dependent arc multiplicity, marking-dependent firing rates, and reward rates defined at the net level.

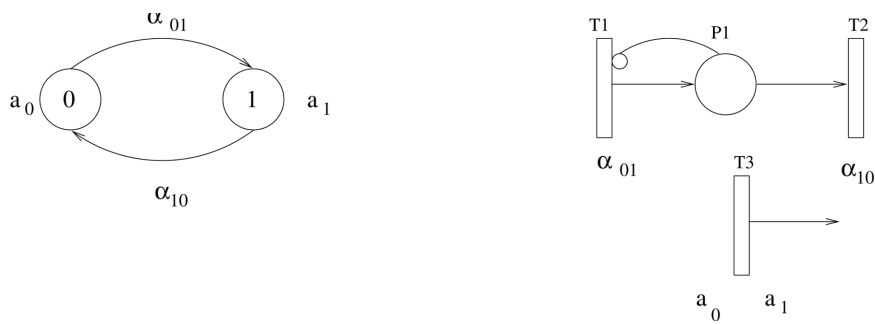
**Priorities:** As mentioned in the previous section, priority is important when more than one transition is enabled at the same time. Although inhibitor arcs can be used to achieve priority relationships, for the purpose of simplifying the model description, explicit priorities can be assigned to transitions. Priorities are specified by assigning integer numbers to transitions. A transition is enabled only if there is no other transition with a higher priority enabled.

**Guards:** The guard functions are similar to the inhibitor arcs, but can use the entire state of the net rather than just the number of tokens in places. They determine when transitions are to be enabled. This feature provides a powerful means to simplify the graphical representation and to make SRNs easier to understand in a more general way compared to the use of inhibitor arcs.

**Marking-Dependent Arc Multiplicity:** This feature provides a way to change the structure of SRNs. For example, when a critical component of the system is down, the system is down. The way for us to represent the situation is to flush all places which have number of tokens representing available resources in the system. The example showing the use of this feature is in the section 2.4.5.

**Marking-Dependent Firing Rates:** The firing rate of a transition may depend in a rather general way based on the current marking of the net. In the implementation, there are two ways: one way is to use rate functions, which are similar to guard functions and reward

rate functions; another way is to use the number of tokens in a chosen place multiplying the basic rate of the transition, which is called place-dependent firing rate. For the first situation, there is a SRN example of Markov Modulated Poisson Processes (MMPPs) [4] in the right part of Figure 2.5. The firing rate of the transition  $T3$  depends on whether there is a token in the place  $P1$ . When there is one and only one token in  $P1$ , the firing rate is  $a_1$ ; Otherwise, it is  $a_0$ . Since there is an inhibitor arc from  $P1$  to  $T1$ ,  $P1$  can only have one token at most.



**Figure 2.5:** Extension of GSPN 2

The left part of Figure 2.5 shows the corresponding CTMC which decides the firing rate of the transition  $T3$ .

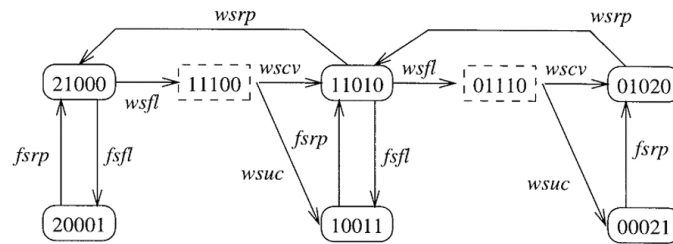
**Reward Rate Specification:** The basic output measures obtained from a SRN are the throughput of a transition and the mean number of tokens in a place. But that's far from enough. Normally, more general information, such as the probability that a place is empty while another one is full, or the sum of the number of tokens in a set of places, is necessary. Since it is at the net level rather than at the place level, reward rate functions are introduced.

Compared to GSPN, SRN provides more power and eases the work of translating real-world systems into analytic models. That's why SRN has been implemented in SHARPE.

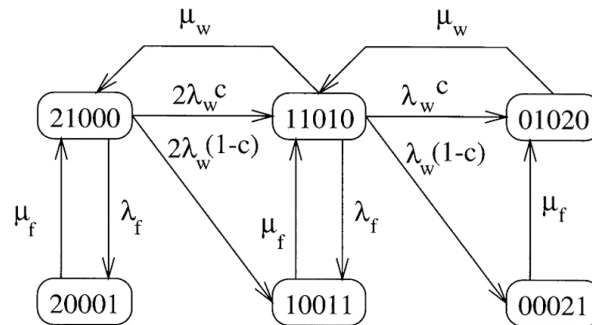
## 2.2 How to solve

First, consider a computing system model (example 2.4.1) shown in Figure 2.8.

Next step, the SRN in Figure 2.8 is converted into the corresponding reachability graph. Figure 2.6 shows the reachability graph. Notice that vanishing markings are shown as dotted rectangle.



**Figure 2.6:** The Reachability Graph for the system in Figure 2.8



**Figure 2.7:** CTMC after deleting vanishing markings from Figure 2.6

Assign rates and probabilities to each arc in the reachability graph, and eliminate all

vanishing markings. The corresponding Continuous Time Markov Chain is shown in Figure 2.7, where, respectively,  $\lambda_w$  and  $\lambda_f$  are the failure rates of each workstation and the file server, and  $\mu_w$  and  $\mu_f$  represent the repair rates of each workstation and the file server.

For transient analysis, *randomization* [15], sometimes called *uniformization*, is used to solve the problem. For steady-state analysis, Gauss-Seidel and Successive Over-Relaxation are used.

## 2.3 Implementation

### 2.3.1 Syntax for SRNs

The syntax for SRN models in SHARPE is as the following:

```

srn name {(param_list)}
* section 1: places and initial numbers of tokens
<place_name expression>
end
* section 2: timed transition names, types and rates
{
<transition_name ind expression {guard expression } {priority expression}>
<transition_name placedep place_name expression {guard expression } {priority
expression}>
<transition_name gendep expression {guard expression } {priority expres-
sion} >
}
end
* section 3: immediate transition names, types and weights

```

```

{
<transition_name ind expression{guard expression } {priority expression}>
<transition_name placedep place_name expression{guard expression } {priority
expression}>
<transition_name gendep expression {guard expression } {priority expres-
sion} >
}
end
* section 4: place-to-transition arcs and multiplicity
{ <place_name transition_name expression> }
end
* section 5: transition-to-place arcs and multiplicity
{ <transition_name place_name expression> }
end
* section 6: inhibitor arcs and multiplicity
{ <place_name transition_name expression> }
end

```

where, *param\_list* is:

*name, name, ..., name*

*name*, *trans\_name* and *place\_name* are all symbols; *expression* is a mathematical expression that could contain function calls; *ind* means that the transition's firing rate is not dependent on the current marking of the net; *placedep* means that the transition's firing rate depends on the number of tokens in the specific place mentioned and the expression assigned to it; and *gendep* means that the firing rate depends on the marking-dependent function referenced in the corresponding *expression*.



## 2.3.2 New built-in functions

### Marking-dependent and rate-dependent functions

The following functions are used only within reward functions, guard functions, rate functions, and arc cardinality functions for SRN models.

- $\#(place\_name)$

Returns the number of tokens in a place with the given *place\_name*.

- $?(trans\_name)$

Returns the boolean (true or *false*) value depending on whether the given transition *trans\_name* is enabled.

- $Rate(trans\_name)$

Returns the rate of the given transition *trans\_name*; if disabled, return 0.

### System analysis functions

In addition to the system analysis functions used for GSPN, three new system analysis functions have been introduced to deal with the power of SRN models.

- $srn\_extrss(sys\_name ; reward\_func\_name\{; arglist\})$

Calculates the steady-state expected value of the reward function *reward\_func\_name*.

- $srn\_extrt(t, sys\_name; reward\_func\_name\{; arglist\})$

Calculates the expected value of the reward function *reward\_func\_name* at time *t*.

- **srn\_cexrt** ( $t, sys\_name; reward\_func\_name\{; arglist\}$ )

Calculates the cumulative expected value of the reward function  $reward\_func\_name$  over the interval  $(0, t]$ .

- **srn\_ave\_cexrt** ( $t, sys\_name; reward\_func\_name\{; arglist\}$ )

Calculates the average cumulative expected value of the reward function  $reward\_func\_name$  over the interval  $(0, t]$ .

- **mtta** ( $sys\_name \{; arglist\}$ )

Calculates the mean time to absorption for the SRN named  $sys\_name$ . The function should be used only when the underlying CTMC has absorbing states. (See example C.4.1)

- **srn\_cexrinf** ( $sys\_name; reward\_func\_name\{; arglist\}$ )

Calculates the cumulative expected value of the reward function  $reward\_func\_name$  until absorption for the corresponding CTMC of the SRN system  $sys\_name$ . The CTMC must have absorbing states. (See example C.4.1)

where,  $arglist$  is

$expression, expression, \dots, expression$

## Mathematical functions

All the following functions can be used within *expressions*, for all models including the SRN model.

- **acos** ( $expression$ )

Calculates the arccosine.

- **asin** (*expression*)

Calculates the arcsine.

- **atan** (*expression*)

Calculates the arctangent.

- **ceil** (*expression*)

Calculates the ceiling of a value.

- **cos** (*expression*)

Calculates the cosine.

- **fabs** (*expression*)

Calculates the absolute value.

- **floor** (*expression*)

Calculates the floor of a value.

- **ln** (*expression*)

Calculates natural logarithm.

- **max** (*expression, expression*)

Compares two values and returns the larger one.

- **min** (*expression, expression*)

Compares two values and returns the smaller one.

- **sin** (*expression*)

Calculates sine.

- **sqrt** (*expression*)

Finds square root.

- **tan** (*expression*)

Calculates the tangent.

- **weibull** (*expression1*, *expression2*, *expression3*)

Calculates the Weibull distribution function  $1 - e^{-\frac{\text{expression1} \times \text{expression3}}{\text{expression2}}}$

### 2.3.3 Syntax extensions

#### User defined function

Now, SHARPE supports either the old way of defining a function:

**func** (*param\_list*) *expression*

or the new way:

**func** (*param\_list*)

<statement>

**end**

**If**-statement has been added:

**if** *bool\_expression*

<statement>

{ < **elseif** *bool\_expression*

<statement> >}

```
{else  
  
<statement>}  
  
end
```

where *statement* can be

```
expression | bind var_name expression | epsilon epsilon_type expression | if statement
```

Detailed examples are provided in section 2.4.

### Fixed point iteration

Suppose we have one SRN model  $M_1$ . The firing rate  $R_1$  of a transition  $T_1$  is the same as the throughput of another transition  $T_2$  [5]. Since we don't know the firing rate of  $T_1$ , fixed point iteration has to be used:

1. Set error bound  $e$  as a small real number, normally  $1e - 7$  in SHARPE.
2. Initialize the firing rate  $R_1^0$  of  $T_1$  to a reasonable value.
3. Set  $k = 1$
4. Execute  $M_1$ , compute the throughput  $T_{2_{throughput}}$  of  $T_2$ .
5. Set  $R_1^k = T_{2_{throughput}}$ .
6. If  $|R_1^k - R_1^{k-1}|/R_1^{k-1} < e$ , then stop, else set  $k = k + 1$  and goto step 4.

Under a very general condition, the solution always exists, but the uniqueness of the solution is not guaranteed [2]. However, in many of practical problems, result is often unique, so the justification is enough for the practical use of fixed point iterations.

To support fixed point iteration, **while**-statement has been introduced:

**while** *bool\_expression*

<statement>

**end**

where *statement* can be

**expr** *expression*{*expression* ...} | **bind** *var\_name* *expression* | **epsilon** *epsilon\_type*  
*expression* | **if\_statement** | *loop* | **while\_statement**

There is an example of fix-point iteration in section 2.4.9. Also, an example of **while**-statement has been included in section 2.4.10.

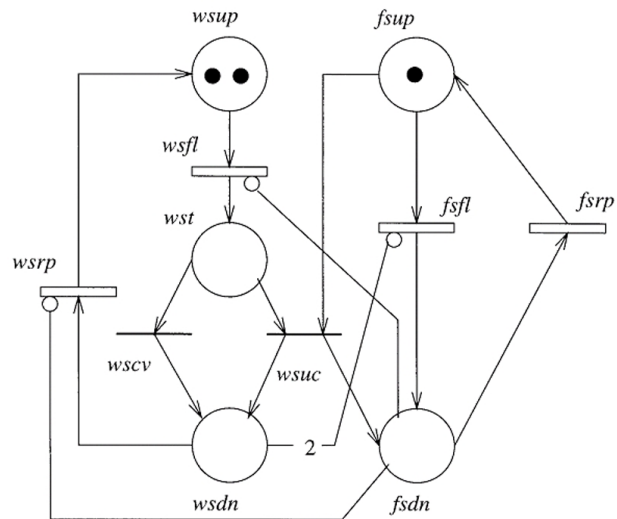
## 2.4 SRN Examples

### 2.4.1 Two workstations, one file server system

#### Description

A system contains 2 workstations and 1 file server (Figure 2.8) . Suppose the network is fault-free, and the whole system is working as long as there is one workstation and the file server is operational. So, the initial number of tokens in the place *wsup* is 2 and in the place *fsup* is 1. The file server has higher repair priority than the two workstations(see the inhibitor arc from the place *fsdn* to the transition *wsrp* in Figure 2.8 ). Also, when the whole system is down, currently operational workstations or file server don't go down any more(see the inhibitor arcs from *fsdn* and *wsdn* to the transitions *wsfl* and *fsrp* in Figure 2.8). We also have the assumption that, when a workstation fails, with probability *c*, the failure is not detected, leading to the corruption and the failure of the file-server. That's

why we have immediate transitions  $wscv$  and  $wsuc$ .



**Figure 2.8:** Two workstation, one file server system with non-perfect failure detect

## Features

- Reward function to compute expected values.
- Transient analysis

**SHARPE File** — *srn/wfs.txt*

format 8

func avail()

if ((#(wsup) > 0) and (#(fsup) == 1))

1

```
else
0
end
end
```

```
srn wfs (c)
```

```
* Places
```

```
wsup 2
```

```
fsup 1
```

```
wst 0
```

```
wsdn 0
```

```
fsdn 0
```

```
end
```

```
* Timed transitions
```

```
wsfl placedep wsup 0.0001
```

```
fsfl ind 0.00005
```

```
wsrp ind 1.0
```

```
fsrp ind 0.5
```

```
end
```

```
* Immediate transitions
```

```
wscv ind c
```

```
wsuc ind 1 -c
```

```
end
```

```
* Input arcs
```

```
wsup wsfl 1
```

```
fsup fsfl 1
```

```
fsup wsuc 1
```

```
wst wscv 1
```

```
wst wsuc 1
```

```
wsdn wrsp 1
```

```
fsdn fsrp 1
```



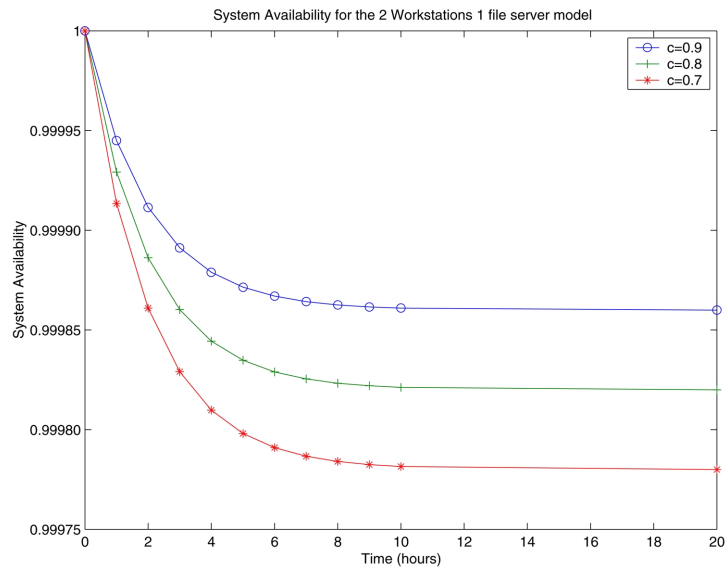
```

end
* Output arcs
wsfl wst 1
wsrp wsup 1
fsfl fsdn 1
fsrp fsup 1
wscv wsdn 1
wsuc wsdn 1
wsuc fsdn 1
end
* Inhibitor arcs
fsdn wsfl 1
fsdn wrsp 1
wsdn fsfl 2
end

* Obtain results
loop c, 0.7, 0.9, 0.1
  loop t, 1, 10, 1
    expr smn_exrt(t, wfs; avail; c)
  end
  expr smn_exrt(20, wfs; avail;c)
end

end

```



**Figure 2.9:** Graph result for example 2.4.1

The result is shown graphically in **Figure 2.9**

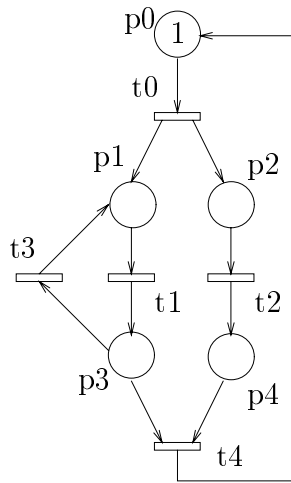
## 2.4.2 Molloy's example

### Source

M. K. Molloy, Performance Analysis Using Stochastic Petri Nets, *IEEE Trans. Comput.*, C-31 (9), Sept. 1982, 913–917.

### Description

The net is shown in Figure 2.10



**Figure 2.10:** SRN for Example 2.4.2

**Features**

- Reward based functions to compute expected values.
- Default measures
- Steady-state analysis

**SHARPE File** — *srn/ex1.txt*

echo M. K. Molloy, Performance Analysis Using Stochastic Petri Nets,  
 echo IEEE Trans. Comput., C-31(9), Sept. 1982, 931-917

format 8

srn example1()

p0 1

p1 0

```
p2 0
p3 0
p4 0
end
t0 ind 1.0
t1 ind 3.0
t2 ind 7.0
t3 ind 9.0
t4 ind 5.0
end
end
p0 t0 1
p1 t1 1
p2 t2 1
p3 t3 1
p3 t4 1
p4 t4 1
end
t0 p1 1
t0 p2 1
t1 p3 1
t2 p4 1
t3 p1 1
t4 p0 1
end
end
```

\* REWARD functions

```
func ef0() #(p0)
```

```
func ef1() #(p1)
```

```
func ef2() Rate(t2)
```

```
func ef3() Rate(t3)
```

```
func eff() Rate(t1)*1.8+#(p3)*0.7
```

```
* Obtain results
```

```
expr srn_exrss(example1; ef0), srn_exrss(example1; ef1), srn_exrss(example1; ef2), srn_exrss(example1; ef3),  
srn_exrss(example1; eff)
```

```
end
```

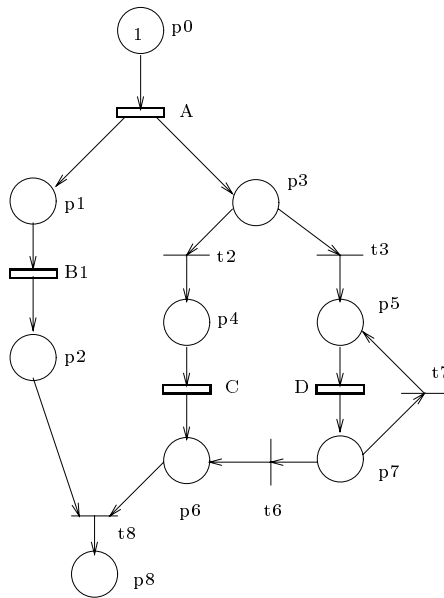
## 2.4.3 Software Performance Analysis

### Description

This example models the following piece of software:

```
A: Statements;  
PARBEGIN  
  B1: statements;  
  B2: IF (cond1) THEN  
    C: statements;  
  ELSE  
    DO  
      D: statements;  
    WHILE (cond2);  
  END IF  
PAREND
```

The corresponding SRN model is shown in Figure 2.11.



**Figure 2.11:** SRN for Example 2.4.3

## Features

- Probability and rate functions.
- Priorities for immediate transitions.
- Reward functions.
- Transient analysis with multiple time points.

## SHARPE File — *srn/ex2.txt*

```

echo Software Performance Analysis
echo A: Statements;
echo PARBEGIN
echo   B1: statements;
echo   B2: IF (cond1) THEN
echo     C: statements;

```

```
echo    ELSE
echo    DO
echo    D: statements;
echo    WHILE (cond2);
echo    END IF
echo PAREND
```

```
format 8
```

```
bind
rate0  1.0
rate1  0.3
prob2  0.4
prob3  0.6
rate4  0.2
rate5  7.0
prob6  0.05
prob7  0.95
prob8  1.0
end
```

```
srn ex2()
```

```
* Places
```

```
P0  1
P1  0
P2  0
P3  0
P4  0
P5  0
P6  0
P7  0
P8  0
```

```
end
* Timed transitions
A ind rate0
B1 ind rate1
C ind rate4
D ind rate5
end
* Immediate transitions
t2 ind prob2
t3 ind prob3
t6 ind prob6
t7 ind prob7
t8 ind prob8
end
* Input arcs
P0 A 1
P1 B1 1
P3 t2 1
P3 t3 1
P4 C 1
P5 D 1
P7 t6 1
P7 t7 1
P2 t8 1
P6 t8 1
end
* Output arcs
A P1 1
B1 P2 1
t2 P4 1
t3 P5 1
C P6 1
```



```

D P7 1
t6 P6 1
t7 P5 1
A P3 1
t8 P8 1
end
* Inhibitor arcs
end

func rfunc() #(P8)

echo probability of completion
loop i, 1, 10
    srn_exrt(i, ex2; rfunc)
end
loop i, 10, 20, 2
    srn_exrt(i, ex2; rfunc)
end
loop i, 20, 50, 5
    srn_exrt(i, ex2; rfunc)
end

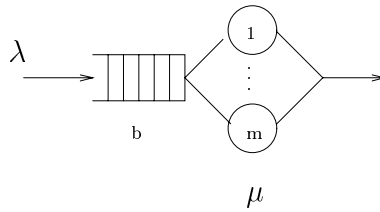
end

```

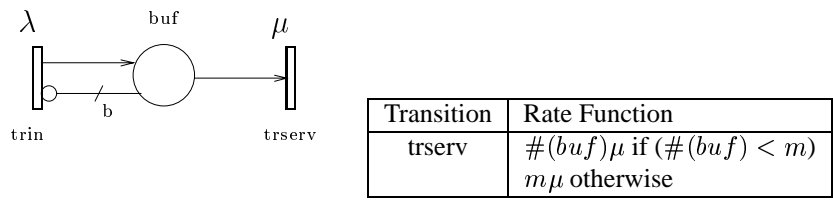
#### 2.4.4 $M/M/m/b$ queue

##### Description

This example models a finite-buffer  $M/M/m/b$  queue shown in Figure 2.12. The corresponding SRN is shown in Figure 2.13.



**Figure 2.12:** The  $M/M/m/b$  Queue.



**Figure 2.13:** SRN for Example 2.4.4

**Features**

- Both steady-state and transient analysis.
- Marking dependent firing rates.
- Reward functions.

**SHARPE File** — *srn/ex3.txt*

echo M/M/m/b queue model

format 8

bind

lambda 0.90

mu 0.10

\*number of buffers

```

b 2
*number of servers
m 2
end

* RATE function
func rate_serv()
if (#(buf) < m)
#(buf)*mu
else
m*mu
end
end

srn example3()
* Places
buf 0
end
* Timed transitions
trin ind lambda
trserv gendep rate_serv()
end
* Immediate transitions
end
* Input arcs
buf trserv 1
end
* Output arcs
trin buf 1
end
* Inhibitor arcs
buf trin b

```

```
end

* REWARD functions
func qlength1() #(buf)

func util1() ?(trserv)

func tput1() Rate(trserv)

func probrej()
if #(buf) == b)
1
else
0
end
end

func probempty()
if #(buf)==0)
1
else
0
end
end

func probhalffull()
if #(buf) == b/2)
1
else
0
end
end
```

```

* Obtain results
expr srn_exrss(example3; qlength1), srn_exrss(example3; tput1), srn_exrss(example3; util1), srn_exrss(example3;
probrej), srn_exrss(example3; probempty), srn_exrss(example3; probhalffull)

loop t, 0.1, 1.0, 0.1
expr srn_exrt(t, example3; qlength1), srn_exrt(t, example3; tput1), srn_exrt(t, example3; util1), srn_exrt(t, ex-
ample3; probrej), srn_exrt(t, example3; probempty), srn_exrt(t, example3; probhalffull)
end

loop t, 1.0, 10.0, 1.0
expr srn_exrt(t, example3; qlength1), srn_exrt(t, example3; tput1), srn_exrt(t, example3; util1), srn_exrt(t, ex-
ample3; probrej), srn_exrt(t, example3; probempty), srn_exrt(t, example3; probhalffull)
end

end

```

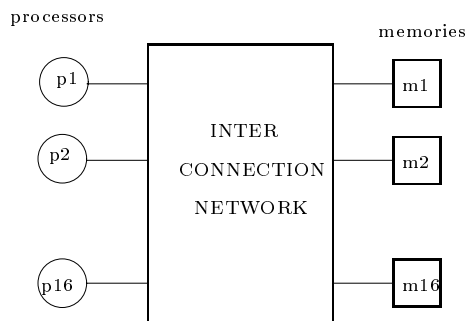
## 2.4.5 C.mmp system performability analysis

### Source

J. T. Blake, A. L. Reibman and K. S. Trivedi, Sensitivity Analysis of Reliability and Performability Measures for Multiprocessor Systems, *Proc. 1988 ACM SIGMETRICS*, Santa Fe, NM, 1988.

### Description

This example models the C.mmp system designed at CMU. The architecture of the system is shown in Figure 2.14. The corresponding SRN model is shown in Figure 2.15.



**Figure 2.14:** The C.mmp Architecture.

## Features

- Guard functions.
- Variable multiplicity arcs.
- Reward based measures.
- Transient analysis.

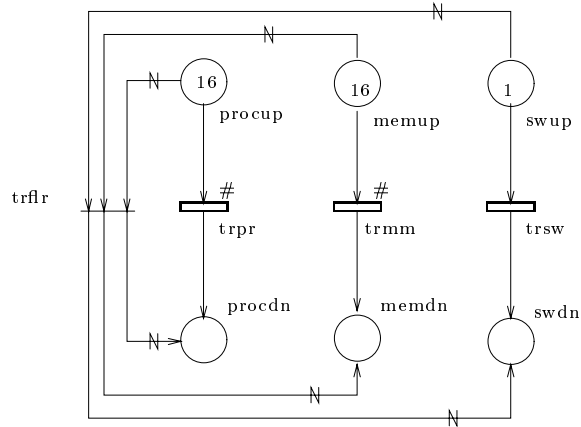
## SHARPE File — *srn/ex4.txt*

```
echo C.mmp system performability analysis
echo J.T. Blake, A.L. Reibman and K.S. Trivedi,
echo Sensitivity Analysis of Reliability and Performability
echo Measures for Multiprocessor Systems,
echo Proc. 1988 ACM SIGMETRICS, Santa Fe, NM, 1988
```

```
format 8
```

```
* Munimum number of proc/mem needed  $1 \leq k \leq 16$ 
```

```
bind k 2
```



Transition	Guard Function
trflr	$((\#(procup) < k) \vee (\#(memup) < k) \vee (\#(swup) = 0))$ $\wedge ((\#(procup) > 0) \vee (\#(memup) > 0) \vee (\#(swup) > 0))$
Arcs	Multiplicity Function
procup $\rightarrow$ trflr & trflr $\rightarrow$ procdn	$\#(procup)$
memup $\rightarrow$ trflr & trflr $\rightarrow$ memdn	$\#(memup)$
swup $\rightarrow$ trflr & trflr $\rightarrow$ swdn	$\#(swup)$

**Figure 2.15:** SRN for Example 2.4.5.

```

* GUARD function
func entrflr()
if (#(procup) == 0 and #(memup) == 0 and #(swup) == 0)
0
elseif (#(procup) < k or #(memup) < k or #(swup) == 0)
1
else
0
end
end

* ARC CARDINALITY functions
func apfl() #(procup)

```

```

func amfl() #(memup)

func asfl() #(swup)

srn example4()
* Places
procup 16
procdn 0
memup 16
memdn 0
swup 1
swdn 0
end
* Timed transitions
trpr placedep procup 0.0000689
trmm placedep memup 0.000224
trsw ind 0.0002202
end
* Immediate transitions
trflr ind 1.0 guard entrflr() priority 100
end
* Input transitions
procup trpr 1
memup trmm 1
swup trsw 1
procup trflr apfl()
memup trflr amfl()
swup trflr asfl()
end
* Output transitions
trpr procdn 1
trmm memdn 1

```



```

trsw swdn 1
trflr procdn apfl()
trflr memdn amfl()
trflr swdn asfl()
end

* Inhibitor arcs
end

* REWARD functions
func reliab()
if (#(procup) ≥ k and #(memup) ≥ k and #(swup) == 1)
1
else
0
end
end

func reward_rate()
if (#(procup) ≥ k and #(memup) ≥ k and #(swup) == 1)
if (#(procup) > #(memup))
bind l #(memup)
bind m #(procup)
else
bind m #(memup)
bind l #(procup)
end
end
bind temp (1.0-(1.0/m))^l
m*(1.0 - temp)
else
0
end
end
end

```

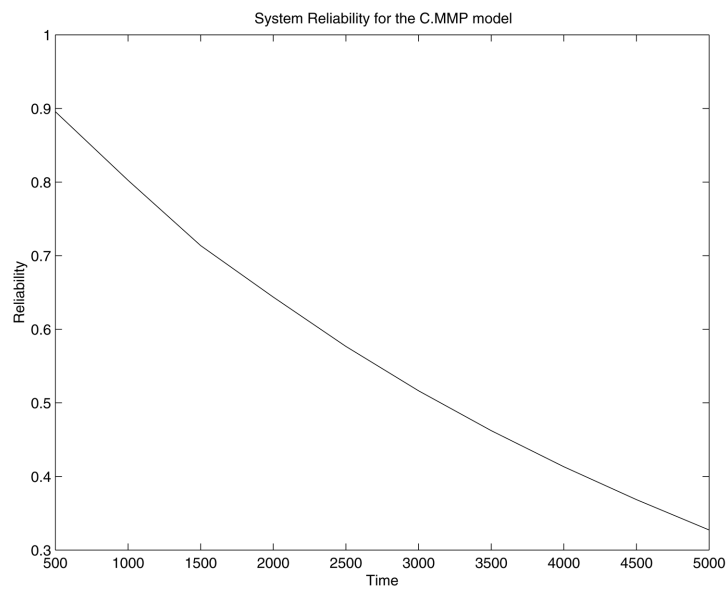
```

* Obtain results
loop t, 500.0, 5000.0, 500.0
  expr  srn_exrt(t, example4; reliab), srn_exrt(t, example4; reward_rate), srn_cexrt(t, example4; reward_rate)
end

end

```

**Result (Figure 2.16)**



**Figure 2.16:** Graph result for example 2.4.5

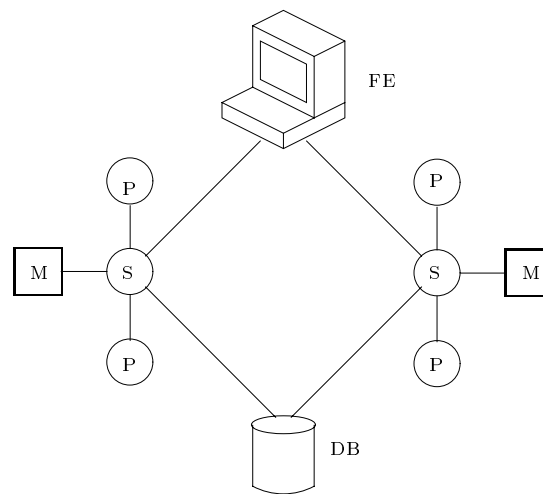
## 2.4.6 Database system availability analysis

### Source

P. Hiedelberger and A. Goyal, Sensitivity Analysis of Continuous Time Markov chains using Uniformization, *Computer Performance and Reliability*, G. Iazeolla, P. J. Courtois and O. J. Boxma (Eds.), Elsevier Science Publishers, B.V. (North-Holland), Amsterdam, 1988.

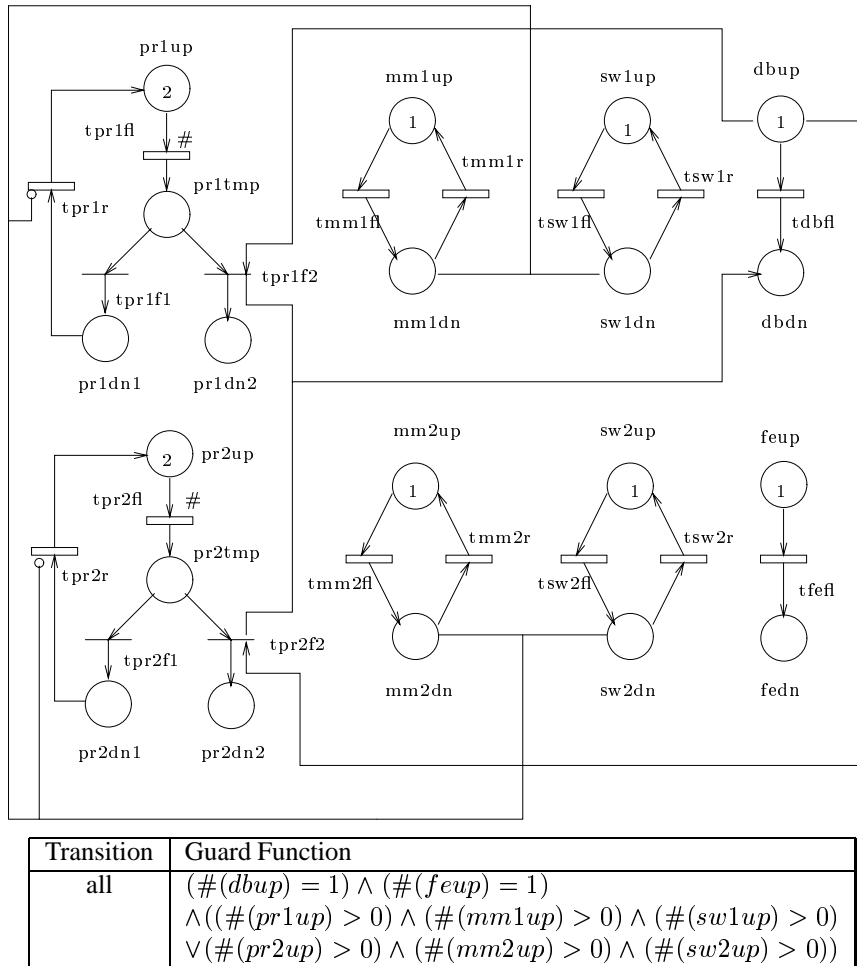
### Description

This example is a model of a database system shown in Figure 2.17.



**Figure 2.17:** The Database System Architecture.

The system consists of a front end (FE), a database (DB) and two processing sub-systems. Each processing sub-system consists of two processors (P), a memory (M) and a switch (S). For the system to be functional, we need at least one of the processing sub-systems to be operational. The database and the front-end should also be operational. The



**Figure 2.18:** SRN for Example 2.4.6.

processing sub-system is functional as long as the memory, the switch and at least one of the processors is functional. When a processor fails, with probability  $c$  it fails without disturbing the system. However, with probability  $1 - c$  the failing processor corrupts the database causing it to fail and consequently rendering the system un-operational. The processors, memories and switches can be repaired while the system is up. The memories and switches receive priority over the processors for repair. The corresponding SRN model is shown in Figure 2.18.

## Features

- Guard function.
- Reward based functions.
- Transient analysis.

### SHARPE File — *srn/ex5.txt*

echo Database system availability analysis

echo P. Hiedelberger and A. Goyal,

echo Sensitivity Analysis of Continuous Time Markov chains using Uniformization,

echo Computer Performance and Reliability, G. Iazeolla, P. J. Courtois and

echo O. J. Boxma (Eds.), Elsevier Science Publishers, B.V. (North–Holland),

echo Amsterdam, 1988

format 8

epsilon basic 1.0e-10

bind coverage 0.99

bind count 0

\* GUARD functions

func enall()

if (#(dbup)==0)

0

elseif (#(feup)==0)

0

elseif (((#(mm1up)==0) or #(sw1up)==0) or #(pr1up)==0) and ((#(mm2up)==0) or #(sw2up)==0) or #(pr2up)==0)))

0

else

```
1
end
end

srn example5()
* Places
* First processing subsystem
mm1up 1
sw1up 1
pr1up 2
mm1dn 0
sw1dn 0
pr1tmp 0
pr1dn1 0
pr1dn2 0
* Second processing subsystem
mm2up 1
sw2up 1
pr2up 2
mm2dn 0
sw2dn 0
pr2tmp 0
pr2dn1 0
pr2dn2 0
* Database
dbup 1
dbdn 0
* Frontend
feup 1
fedn 0
end
* Timed transitions
```

```

tmm1fl ind 1000./2400. guard enall()
tsw1fl ind 1000./2400. guard enall()
tpr1fl placedep pr1up 1000./2400. guard enall()
tmm1r ind 1000. guard enall()
tsw1r ind 1000. guard enall()
tpr1r ind 1000. guard enall()
tmm2fl ind 1000./2400. guard enall()
tsw2fl ind 1000./2400. guard enall()
tpr2fl placedep pr2up 1000./2400. guard enall()
tmm2r ind 1000. guard enall()
tsw2r ind 1000. guard enall()
tpr2r ind 1000. guard enall()
tdbfl ind 1000./2400. guard enall()
tfeff ind 1000./2400. guard enall()
end

* Immediate transitions
tpr1f1 ind coverage priority 100
tpr1f2 ind 1.0-coverage priority 100
tpr2f1 ind coverage priority 100
tpr2f2 ind 1.0-coverage priority 100
end

* Input arcs
mm1up tmm1fl 1
sw1up tsw1fl 1
pr1up tpr1fl 1
pr1tmp tpr1f1 1
pr1tmp tpr1f2 1
dbup tpr1f2 1
mm1dn tmm1r 1
sw1dn tsw1r 1
pr1dn1 tpr1r 1
mm2up tmm2fl 1

```

```

sw2up tsw2fl 1
pr2up tpr2fl 1
pr2tmp tpr2f1 1
pr2tmp tpr2f2 1
dbup tpr2f2 1
mm2dn tmm2r 1
sw2dn tsw2r 1
pr2dn1 tpr2r 1
dbup tdbfl 1
feup tfefl 1
end
* Output arcs
tmm1fl mm1dn 1
tsw1fl sw1dn 1
tpr1fl pr1tmp 1
tpr1f1 pr1dn1 1
tpr1f2 pr1dn2 1
tpr1f2 dbdn 1
tmm1r mm1up 1
tsw1r sw1up 1
tpr1r pr1up 1
tmm2fl mm2dn 1
tsw2fl sw2dn 1
tpr2fl pr2tmp 1
tpr2f1 pr2dn1 1
tpr2f2 pr2dn2 1
tpr2f2 dbdn 1
tmm2r mm2up 1
tsw2r sw2up 1
tpr2r pr2up 1
tdbfl dbdn 1
tfefl fedn 1

```



```

end

* Inhibitor arcs
mm1dn tpr1r 1
mm2dn tpr1r 1
sw1dn tpr1r 1
sw2dn tpr1r 1
mm1dn tpr2r 1
mm2dn tpr2r 1
sw1dn tpr2r 1
sw2dn tpr2r 1
end

* REWARD function
func reliab()
if (#(dbup)==0)
0.0
elseif (#(feup)==0)
0.0
elseif (((#(mm1up)==0) or #(sw1up)==0) or #(pr1up)==0) and (((#(mm2up)==0) or #(sw2up)==0) or #(pr2up)==0)))
0.0
else
1.0
end
end

* Obtain results
loop t, 0.01, 0.1, 0.01
    expr srn_exrt(t, example5; reliab)
end

*echo error cumulated
loop t, 0.1, 1, 0.1
    expr srn_exrt(t, example5; reliab)

```

end

end

## 2.4.7 ATM network under overload

### Source

Chang-Yu Wang, D. Logothetis, K.S. Trivedi and I. Viniotis, Transient Behavior of ATM Networks under Overloads, *Proceedings of the IEEE INFOCOM 96*, San Francisco, CA, pp. 978-985, March 1996.

### Description

This example models ATM (Asynchronous Transfer Mode) networks under overloads. The SRN is shown in Figure 2.19.

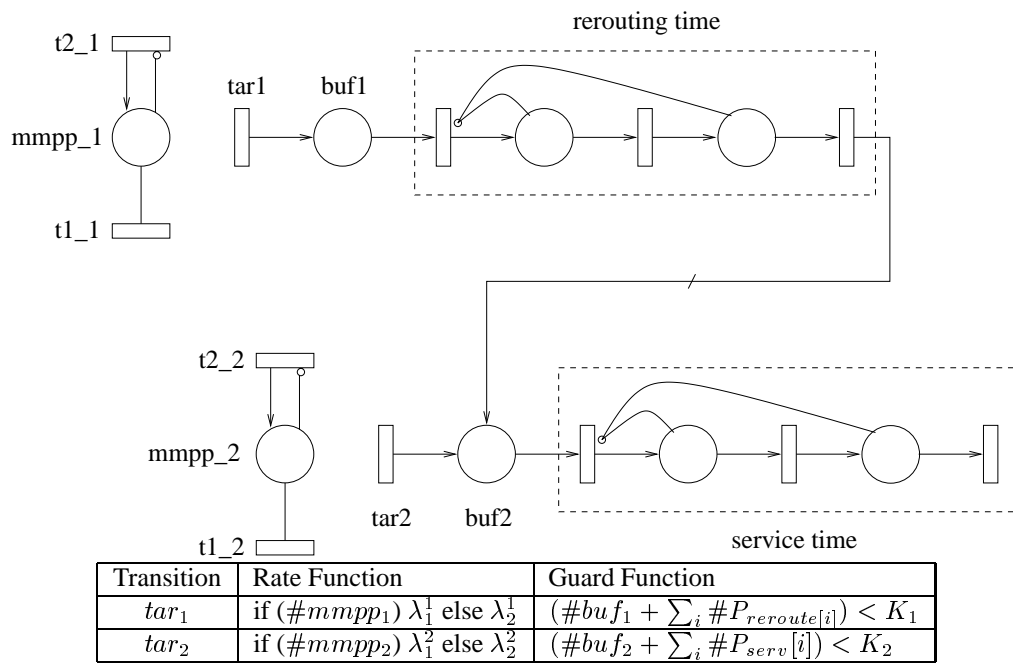
### Features

- Transient analysis.
- Marking dependent firing rates.
- Guard functions.
- Reward functions.

### SHARPE File — *srn/ex6.txt*

echo ATM network under overload

echo Chang–Yu Wang, D. Logothetis, K.S. Trivedi and I. Viniotis,



**Figure 2.19:** SRN for Example 2.4.7

echo Transient Behavior of ATM Networks under Overloads,  
 echo Proceedings of the IEEE INFOCOM 96, San Francisco, CA,  
 echo pp. 978–985, March 1996.

format 8

bind

a1 0.0269163

a2 0.0269163

b1 0.00672908

b2 0.00672908

lambda11 1.5058

lambda21 1.5058

lambda12 0.00301161

lambda22 0.00301161

r1 5

```

r2 5
mu1 2.73
mu2 2.73
K1 16
K2 16
e 0.0001
end

*REWARD Functions
func Qlen1() #(buf1)+#(Er_token1)+#(Er_stage1))/r1

func Earrival()
if #(mmp2)<> 0)
bind ret_val lambda21
else
bind ret_val lambda22
end
if #(Er_token1)==1)
bind ret_val ret_val+r1/mu1
end
ret_val
end

func Qlen2() #(buf2)+#(Er_token2)+#(Er_stage2))/r1

func ELR()
if ((Qlen2()+e)≥K2)
if #(mmp2)<>0)
bind ret_val lambda21
else
bind ret_val lambda22
end

```

```
if (#(Er_token1)==1)
bind ret_val ret_val+r1/mu1
end
ret_val
else
0
end
end
```

```
func PFull()
if (Qlen2()+e)≥K2
1.0
else
0
end
end
```

```
* GUARD Functions
func gar2()
if (Qlen2()+e)<K2
1
else
0
end
end
```

```
func gar1()
if ((Qlen1()+e)<K1)
1
else
0
end
```

```

end

* RATE Functions
func REr1() r1/mu1

func Rar1()
if (#(mmp1_1)>0)
lambda11
else
lambda12
end
end

func REr2() r2/mu2

func Rar2()
if (#(mmp2_2)>0)
lambda21
else
lambda22
end
end

* CARDINALITY Functions
func R2() r2

func dep12()
if ((K2-Qlen2()+e)<1)
0
else
1
end
end

```

```

func R1() r1

srn example6()
* Places
mmpp_1 1
mmpp_2 1
buf1 0
Er_token1 0
Er_stage1 0
buf2 0
Er_token2 0
Er_stage2 0
end
* Timed Transitions
t2_1 ind b1
t2_2 ind b2
t1_1 ind a1
t1_2 ind a2
tar1 gendep Rar1() guard gar1()
Er_trans1 ind REr1()
tar2 gendep Rar2() guard gar2()
Er_trans2 ind REr2()
end
* Immediate Transitions
Er_in1 ind 1. priority 20
Er_out1 ind 1. priority 20
Er_in2 ind 1. priority 20
Er_out2 ind 1. priority 20
end
* Input arcs
mmpp_1 t1_1 1

```

```

mmpp_2 t1_2 1
buf1 Er_in1 1
Er_token1 Er_trans1 1
Er_stage1 Er_out1 R1()
buf2 Er_in2 1
Er_token2 Er_trans2 1
Er_stage2 Er_out2 R2()
end

* Output arcs
t2_1 mmpp_1 1
t2_2 mmpp_2 1
tar1 buf1 1
Er_in1 Er_token1 R1()
Er_trans1 Er_stage1 1
Er_out1 buf2 dep12()
tar2 buf2 1
Er_in2 Er_token2 R2()
Er_trans2 Er_stage2 1
end

* Inhibitor arcs
mmpp_1 t2_1 1
mmpp_2 t2_2 1
Er_token1 Er_in1 1
Er_stage1 Er_in1 1
Er_token2 Er_in2 1
Er_stage2 Er_in2 1
end

* Obtain results
loop t, 10.0, 200.0, 10.0
    expr srn_exrt(t, example6; Qlen1)
    expr srn_exrt(t, example6; Qlen2)

```



```

expr srn_exrt(t, example6; ELR)
expr srn_exrt(t, example6; PFull)
expr srn_exrt(t, example6; Earrival)
end

end

```

### Result (Figure 2.20)

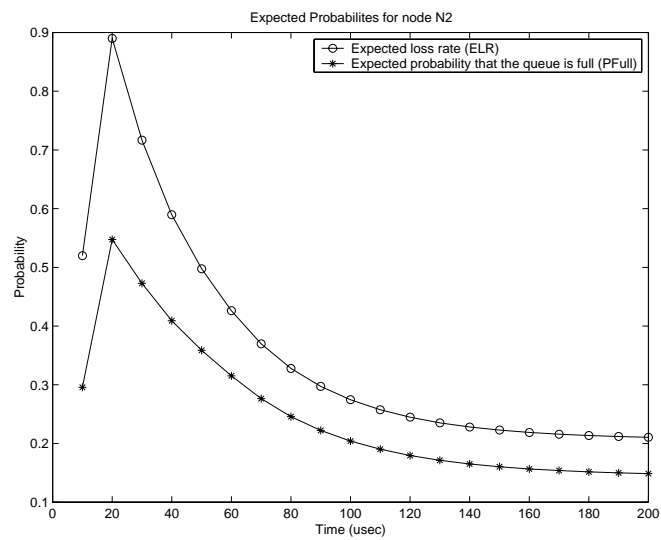


Figure 2.20: Partial graph result for example 2.4.7

## 2.4.8 Criticality Importance and Birnbaum Importance

### Source

R. M. Fricks and K. S. Trivedi, On Computing Importance Measures Using Reward Models, *VII Simposio de Computadores Tolerantes a Falhas (VII SCTF)*, pp. 169 – 183, Campinas Grande, Brazil, Jul. 1997.

## Description

A novel technique for computing importance measures in state space dependability models is introduced here. Specifically, reward functions in a Markov reward model are utilized for this purpose, in contrast to the common method of computing importance measures through combinatorial models and structure functions. The following simple example is used to show how to calculate Criticality Importance and Birnbaum Importance.

## Features

- Reward based measures.

## SHARPE File — *srn/ex7.txt*

```
echo Criticality Importance and Birnbaum Importance
echo R.M. Fricks and K. S. Trivedi,
echo On Computing Importance Measures Using Reward Models,
echo VII Simposio de Computadores Tolerantes a Falhas (VII SCTF),
echo pp. 169–183, Campina Grande, Brazil, Jul. 1997.
```

```
format 8
```

```
* REWARD RATE FUNCTIONS
```

```
* Criticality
```

```
func Q1()
```

```
if (#(p1) == 1)
```

```
1
```

```
else
```

```
0
```

```
end
```

```
end
```

```
func Q2()
```

```
if (#(p2) == 1)
```

```
1
```

```
else
```

```
0
```

```
end
```

```
end
```

```
func Q3()
```

```
if (#(p3) == 1)
```

```
1
```

```
else
```

```
0
```

```
end
```

```
end
```

```
func Q()
```

```
if (Q1() + Q2() + Q3()  $\geq$  2)
```

```
1
```

```
else
```

```
0
```

```
end
```

```
end
```

```
* Birnbaum
```

```
func g11()
```

```
if (1.0+Q2() + Q3()  $\geq$  2)
```

```
1
```

```
else
```

```
0  
end  
end
```

```
func g10()  
if (Q2() + Q3() ≥ 2)  
1  
else  
0  
end  
end
```

```
func g21()  
if (Q1() + 1.0 + Q3() ≥ 2)  
1  
else  
0  
end  
end
```

```
func g20()  
if (Q1() + Q3() ≥ 2)  
1  
else  
0  
end  
end
```

```
func g31()  
if (Q1()+Q2() + 1.0 ≥ 2)  
1  
else
```

```

0
end
end

func g30()
if (Q1()+Q2() ≥ 2)
1
else
0
end
end

srn example7()
* Places
p1 0
p2 0
p3 0
end
* Timed transitions
t1 ind 0.001
t2 ind 0.002
t3 ind 0.003
end
* Immediate transitions
end
* Input arcs
end
* Output arcs
t1 p1 1
t2 p2 1
t3 p3 1
end

```

```
* Inhibitor arcs
```

```
p1 t1 1
```

```
p2 t2 1
```

```
p3 t3 1
```

```
end
```

```
* Obtain results
```

```
bind
```

```
t 20.
```

```
b1 srn_exrt(t, example7; g11) - srn_exrt(t, example7; g10)
```

```
b2 srn_exrt(t, example7; g21) - srn_exrt(t, example7; g20)
```

```
b3 srn_exrt(t, example7; g31) - srn_exrt(t, example7; g30)
```

```
q srn_exrt(t, example7; Q)
```

```
end
```

```
expr b1, b2, b3, b1*srn_exrt(t, example7; Q1)/q, b2*srn_exrt(t, example7; Q2)/q, b3*srn_exrt(t, example7; Q3)/q
```

```
end
```

## 2.4.9 Channel recovery scheme in a cellular network

### Source

Y. Ma, C. W. Ro and K. S. Trivedi, Performability Analysis of Channel Allocation with Channel Recovery Strategy in Cellular Network, *Proceedings of IEEE 1998 International Conference on Universal Personal Communications (ICUPC'98)*, Florence, Italy, 5-9 October, 1998.



echo Communications (ICUPC 1998), Florene, Italy, 5–9 October, 1998.

format 8

bind

MAX\_ITERATIONS 6

MAX\_ERROR 1e-7

t\_channel 28

g\_c 1

\* New call arrival rate

lam\_n 10

\* handoff every 5 minutes

lam\_h\_o 0.33

\* Handoff\_in rate

lam\_h\_i 0.2

\* call duration: 120 seconds

lam\_d 0.5

lam\_f 0.000016677

mu\_r 0.0167

end

srn icupc98 ()

\* Places

T 0

B 0

R 0

CP t\_channel

end

\* Timed transitions

t\_n ind lam\_n

t\_h\_i ind lam\_h\_i

t\_d placedep T lam\_d



```

t_f placedep T lam_f
t_h_o placedep T lam_h_o
t_r ind mu_r
end

* Immediate transitions
t_l ind 1.0 priority 100
end

* Input arcs
CP t_n g_c + 1
CP t_h_i 1
T t_h_o 1
T t_d 1
T t_f 1
R t_r 1
B t_l 1
CP t_l 1
end

* Output arcs
t_n T 1
t_n CP g_c
t_h_i T 1
t_h_o CP 1
t_d CP 1
t_f B 1
t_f R 1
t_r CP 1
t_l T 1
end

* Inhibitor arcs
end

* REWARD rate functions

```

```

func BH()
if (#(CP) == 0)
1.0
else
0.0
end
end
end

```

```

func BN()
if (#(CP) ≤ g_c)
1.0
else
0.0
end
end
end

```

```

func ACh() #(CP)

```

```

func hotput() Rate(t_h_o)

```

```

func ftput2() Rate(t_f)

```

```

func fnum() #(B)

```

```

bind i 0

```

```

bind err 1

```

```

while (i < MAX_ITERATIONS and err > MAX_ERROR)

```

```

bind tp srn_exrss(icupc98; hotput)

```

```

bind err fabs((lam_h_j - tp)/tp)

```

```

bind i i + 1

```

```

if (i < MAX_ITERATIONS)

```

```

bind lam_h_i tp
end
end

expr srn_exrss(icupc98; BH)
expr srn_exrss(icupc98; BN)
expr srn_exrss(icupc98; ACh)
expr srn_exrss(icupc98; fnum)/srn_exrss(icupc98; fput2)

end

```

**Result File** — *srn/ex8.txt.out*

- \* Y. Ma, C. W. Ro and K. S. Trivedi, Performability Analysis of Channel
- \* Allocation with Channel Recovery Strategy in Cellular Network,
- \* Proceedings of IEEE 1998 International Conference on Universal Personal
- \* Communications (ICUPC 1998), Florene, Italy, 5–9 October, 1998.

```

tp <- 4.054972
err <- 0.950678
i <- 1.000000
    lam_h_i <- 4.054972
tp <- 5.557387
err <- 0.270346
i <- 2.000000
    lam_h_i <- 5.557387
tp <- 6.098202
err <- 0.088684
i <- 3.000000
    lam_h_i <- 6.098202
tp <- 6.280690
err <- 0.029055
i <- 4.000000

```

```
lam_h.i <- 6.280690
tp <- 6.340547
err <- 0.009440
i <- 5.000000
lam_h.i <- 6.340547
tp <- 6.359983
err <- 0.003056
i <- 6.000000
```

---

```
srn_exrss(icupc98; BH): 6.50059657e-003
```

---

```
srn_exrss(icupc98; BN): 3.03008702e-002
```

---

```
srn_exrss(icupc98; ACh): 8.70770327e+000
```

---

```
srn_exrss(icupc98; fnum)/srn_exrss(icupc98; ftput2): 4.21143605e-004
```

## 2.4.10 Testing while statement

### Description

This example is used to test the syntax of **while**-statement.

## SHARPE File — *srn/syntaxtest*

```
bind i1 1
```

```
bind a 2
```

```
while i1 ≤ 3
```

```
loop j1, 1, 3, 1
```

```
bind k1 1
```

```
while k1 ≤ 3
```

```
expr i1, j1, k1
```

```
bind k1 k1+1
```

```
end
```

```
end
```

```
bind i1 i1+1
```

```
if a > 1
```

```
loop l1, 1, 3, 1
```

```
expr l1
```

```
end
```

```
end
```

```
end
```

```
loop i2, 1, 3, 1
```

```
bind j2 1
```

```
while j2 ≤ 3
```

```
expr i2, j2
```

```
bind j2 j2+1
```

```
end
```

```
end
```

```
expr min(1, 2), max(1, 2)
```

```
echo ERROR: while cannot be used in func definition
```

```
func test ()  
while a > 1  
end  
end  
  
end
```

# Chapter 3

## Model Types Integrated

### 3.1 Phased-Mission Systems(PMS)

The PMS model is implemented by Xinyu Zang [18], which has the following features:

- An efficient BDD-based algorithm is used for analysis, where BDD stands for binary decision diagrams [8, 1].
- The system configuration in each phase is specified by a fault tree.
- Transient analysis is provided.

#### 3.1.1 Specification of model

The paradigm of fault tree models is used to specify the system configuration in each phase.

A PMS is specified as follows:

```
pms name { (param_list) }  
  <phase_number phase_name duration>  
end
```

The *phase\_number* specifies which phase the system configuration is in. The *phase\_name* should be the same as the *system\_name* in the fault tree in which the system configuration is specified. The *duration* specifies the duration of this phase.

### 3.1.2 System analysis function

The only system analysis function that can be used from PMS model is

**tvalue**( $t$ , *system\_name*)

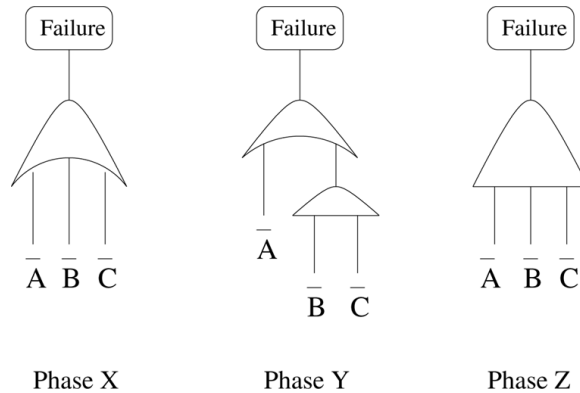
that gives the unreliability of the PMS at time  $t$ . Note that there may be latent faults at the transition of phases. Two switch commands are used to set which time the **tvalue** uses:

- **ltimep**: set time as  $t_-$ , i.e. at the end of the phase  $i - 1$ .
- **rtimep**: set time as  $t_+$ , i.e. at the beginning of the phase  $i$ .

There are two examples included in the next section.

### 3.1.3 Examples

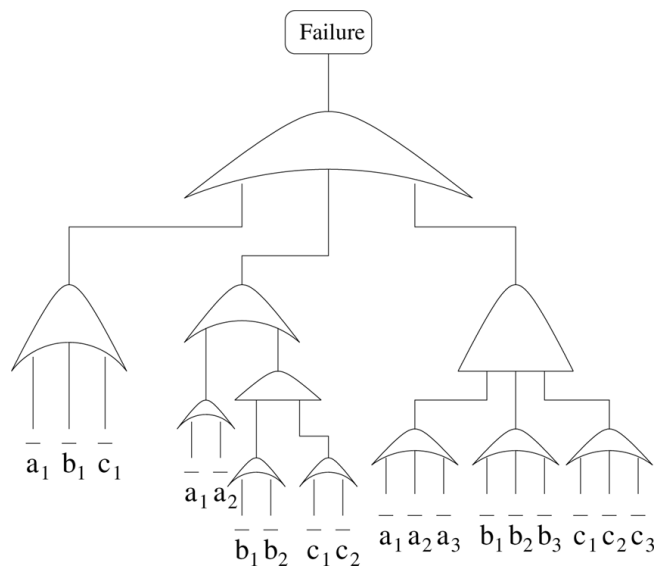
#### A three-phase system



**Figure 3.1:** System configuration in three phases

**Description** The system has three phases  $X$ ,  $Y$  and  $Z$  whose configurations are shown in Figure 3.1 in fault tree format. The equivalent system for the end of mission  $XYZ$  is





**Figure 3.2:** Equivalent system for the end of mission

shown in Figure 3.2. We also consider the other five possible phase configurations, i.e.,  $XZY, YXZ, YZX, ZXY, ZYX$ .

**SHARPE File** — pms/yy.timep

format 8

epsilon results 0.000000000001

ftree X

basic a exp(a.x)

basic b exp(b.x)

basic c exp(c.x)

or top a b c

end

ftree Y

basic a exp(a.y)

```
basic b exp(b.y)
basic c exp(c.y)
and BC b c
or top BC a
end
```

```
ftree Z
basic a exp(a.z)
basic b exp(b.z)
basic c exp(c.z)
and ABC a b c
end
```

```
bind
a_x 0.0001
a_y 0.0001
a_z 0.0001
b_x 0.0001
b_y 0.0001
b_z 0.0001
c_x 0.0001
c_y 0.0001
c_z 0.0001
T_x 10
T_y 10
T_z 10
end
```

```
pms XYZ
1 X T_x
2 Y T_y
3 Z T_z
```

end

pms XZY

1 X T\_x

2 Z T\_z

3 Y T\_y

end

pms YXZ

1 Y T\_y

2 X T\_x

3 Z T\_z

end

pms YZX

1 Y T\_y

2 Z T\_z

3 X T\_x

end

pms ZXY

1 Z T\_z

2 X T\_x

3 Y T\_y

end

pms ZYX

1 Z T\_z

2 Y T\_y

3 X T\_x

end

ltimep

```
loop t, 0, 30, 10
  expr tvalue(t; XYZ), tvalue(t; XZY)
  expr tvalue(t; YXZ), tvalue(t; YZX)
  expr tvalue(t; ZXY), tvalue(t; ZYX)
end
```

```
rtimep
```

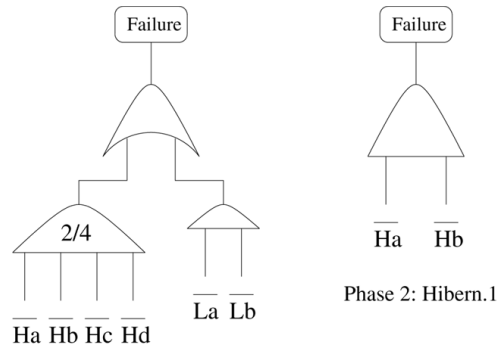
```
loop t, 0, 30, 10
  expr tvalue(t; XYZ), tvalue(t; XZY)
  expr tvalue(t; YXZ), tvalue(t; YZX)
  expr tvalue(t; ZXY), tvalue(t; ZYX)
end
```

```
end
```

## Space application

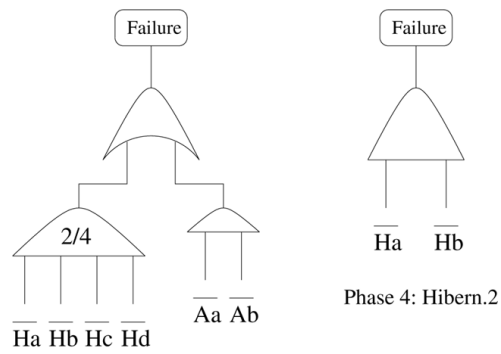
**Description** Modifying the space application in [11], we get an example whose mission alternates between operational phases *Launch*, *Asteroid*, *Comet*, with *Hibernation* phases as shown in Figure 3.3.

**SHARPE File** — pms/space



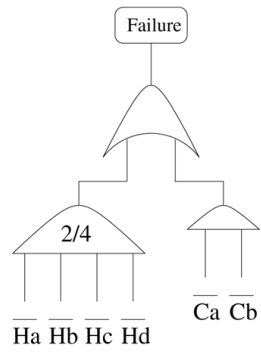
Phase 1: Launch

Phase 2: Hibern.1



Phase 3: Asteroid

Phase 4: Hibern.2



Phase 5: Comet

**Figure 3.3:** System configuration for space application

format 8

\* Phase 1

ftree Launch

repeat La exp(RL)

repeat Lb exp(RL)

repeat Ha exp(RHo)

repeat Hb exp(RHo)

repeat Hc exp(RHo)

repeat Hd exp(RHo)

and L La Lb

kofn H 2,4, Ha Hb Hc Hd

or top L H

end

\* Phase 2

ftree Hibernation1

repeat Ha exp(RHh)

repeat Hb exp(RHh)

and top Ha Hb

end

\* Phase 3

ftree Asteriod

repeat Aa exp(RA)

repeat Ab exp(RA)

repeat Ha exp(RHo)

repeat Hb exp(RHo)

repeat Hc exp(RHo)

repeat Hd exp(RHo)

and A Aa Ab

kofn H 2,4, Ha Hb Hc Hd

or top A H  
end

\* Phase 4

ftree Hibernation2  
repeat Ha exp(RHh)  
repeat Hb exp(RHh)  
and top Ha Hb  
end

\* Phase 5

ftree Comet  
repeat Ca exp(RC)  
repeat Cb exp(RC)  
repeat Ha exp(RHo)  
repeat Hb exp(RHo)  
repeat Hc exp(RHo)  
repeat Hd exp(RHo)  
and C Ca Cb  
kofn H 2,4, Hd Hc Hb Ha  
or top C H  
end

bind

RL 0.00005  
RA 0.00001  
RC 0.0001  
RHo 0.00001  
RHh 0.000001  
T1 48  
T2 17520  
T3 672

T4 26952

T5 672

end

pms Space

1 Launch T1

2 Hibernation1 T2

3 Asteriod T3

4 Hibernation2 T4

5 Comet T5

end

loop t, T1+T2+T3+T4, T1+T2+T3+T4+T5, 112

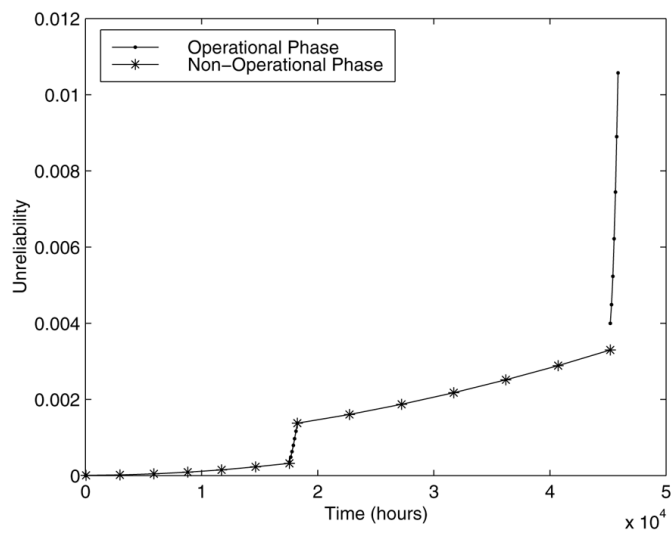
expr tvalue(t; Space)

end

end

**Result** Unreliability of space application





**Figure 3.4:** Unreliability of space application

## 3.2 Multistate Fault Trees

The Multi-state Fault Tree (MFT) model [18] is added in SHARPE as a new model that has the following features:

- An efficient BDD-based analysis algorithm is used for the MFT solution.
- The specification of MFT model is an extension of fault tree model.
- Most types of results for fault tree model are supported.

### 3.2.1 Specification of model

A multi-state fault tree is specified by the following:

```
mstree name { (param_list) }  
<mstreeline>  
end
```

An *mstreeline* has one of the following forms:

1. **basic** *name:state ep*

This is a basic component type. It is assigned a name, a state and an exponential polynomial. Whenever this name appears later in the multi-state tree specification, it is interpreted as being the same state of the same physical component.

2. **transfer** *name name*

The second name must have been previously defined using **basic**. Whenever the first name appears later in the multi-state tree specification, it is interpreted as being the same physical component as the second name.

3. **and** *name name{:state} name{:state} { name{:state} ... }*

This represents an “and” gate. The gate is assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs.

4. **or** *name name{:state} name{:state} { name{:state} ... }*

This represents an “or” gate. The gate is assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs.

5. **kofn** *name expression, expression, name{:state}*

This represents a *k*-out-of-*n* gate having identical inputs. The gate is assigned the first name. The first expression gives *k* and the second expression gives *n*. The inputs to the gate are assumed to be *n* identically distributed, independent copies of the second name.

6. **kofn** *name expression, expression, name{:state} name{:state} { name{:state} ... }*

This represents a *k*-out-of-*n* gate whose inputs need not be identical. The gate is assigned the first name. The first expression gives *k* and the second expression gives *n*. The names following the second expression are the inputs to the gate; there must be at least two.

In forms 2 through 6, the names making up the block must already be defined. The block names that are *top:state* represent a state of top event in multi-state tree.

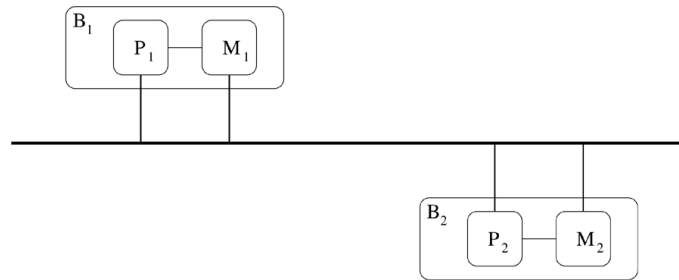
### 3.2.2 System analysis functions

Most types of results for fault tree model are supported, except for importance measure and mincuts. A state of top event (*top:state*) needs to be specified at *state\_eword* in the corresponding functions. For example, if the **cdf** is asked for a state of top event, 1, in a

multi-state tree,  $mst$ ,  $\mathbf{cdf}(mst, top:1)$  can give the result. Detailed description of fault tree models can be found in [14].

### 3.2.3 Examples

#### Two boards system



**Figure 3.5:** System diagram

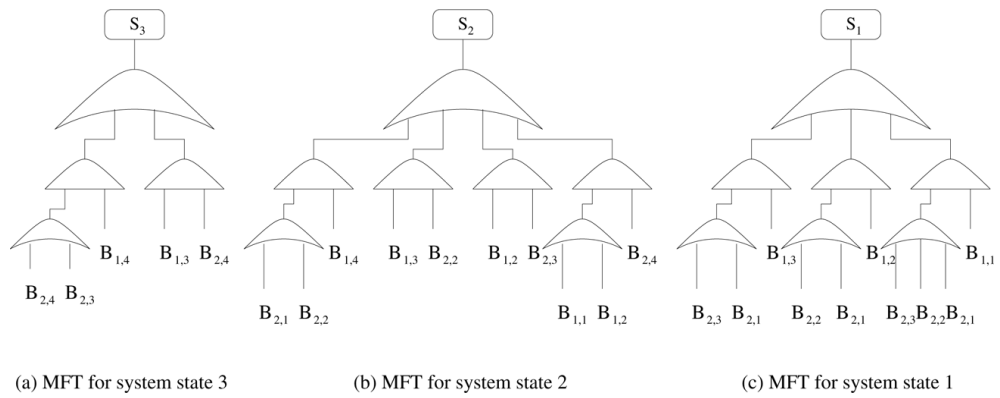
**Description** Figure 3.5 shows a system with two boards  $B_1$  and  $B_2$ , each having a processor and a memory. The memories ( $M_1$  and  $M_2$ ) can be shared by both processors ( $P_1$  and  $P_2$ ). The processor and memory on the same board can fail separately, but  $s$ -dependently. We define system state as: **state 1**, no processor or no memory are functional; **state 2**, at least one processor and exactly one memory are functional; **state 3**, at least one processor and both of the memories are functional. Figure 3.6 shows the MFTs for all the states of the system, where  $B_{ij}$  represents the board  $B_i$  being in **state j**.

**SHARPE File** — ms/ex1

format 8

mstree ex1

basic B1:4 prob(0.95)



**Figure 3.6:** MFTs of example 3.2.3

basic B1:3 prob(0.02)  
 basic B1:2 prob(0.02)  
 basic B1:1 prob(0.01)  
 basic B2:4 prob(0.95)  
 basic B2:3 prob(0.02)  
 basic B2:2 prob(0.02)  
 basic B2:1 prob(0.01)  
 or gor321 B2:3 B2:4  
 and gand311 B1:4 gor321  
 and gand312 B1:3 B2:4  
 or top:3 gand311 gand312  
 or gor221 B1:1 B1:2  
 or gor222 B2:1 B2:2  
 and gand211 B1:4 gor222  
 and gand212 B1:3 B2:2  
 and gand213 B1:2 B2:3  
 and gand214 gor221 B2:4  
 or top:2 gand211 gand212 gand213 gand214  
 or gor121 B2:3 B2:1  
 or gor122 B2:2 B2:1

```

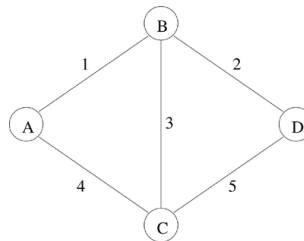
or gor123 B2:3 B2:2 B2:1
and gand111 B1:3 gor121
and gand112 B1:2 gor122
and gand113 B1:1 gor123
or top:1 gand111 gand112 gand113
end

expr sysprob(ex1, top:1)
expr sysprob(ex1, top:2)
expr sysprob(ex1, top:3)

end

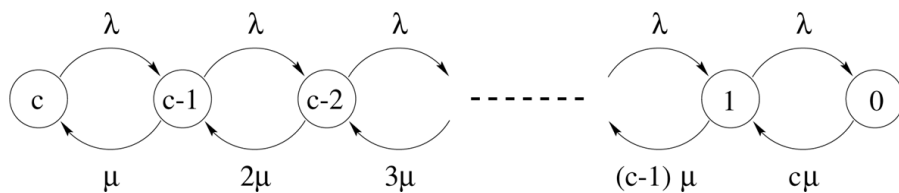
```

### A communication network



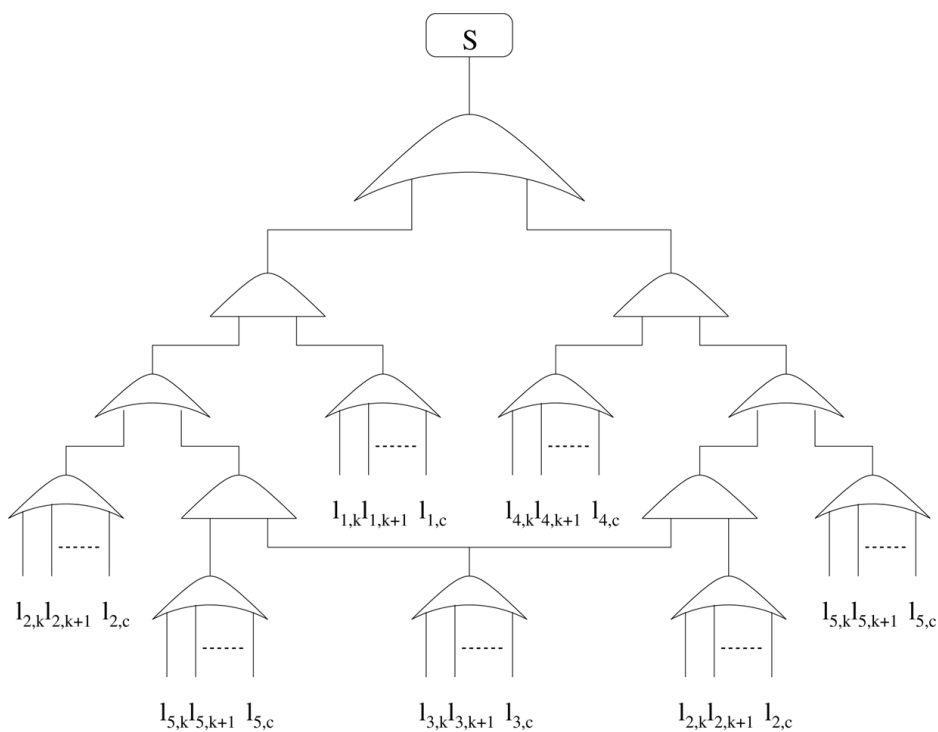
**Figure 3.7:** The network topology of example 3.2.3

**Description** Figure 3.7 shows a communication network topology. Each link can support  $c$  calls/connections simultaneously and the amount of bandwidth required by each call/connection is equal, which means the call/connections are homogeneous. Obviously, the spare capacity of each link has multiple states:  $0, 1, \dots, c$ . We assume the transitions among the states form a birth-death process with parameter  $\lambda$  and  $\mu$  represented as a Con-



**Figure 3.8:** The CTMC for each link's spare capacity in example 3.2.3

tinuous Time Markov Chain (CTMC) in Figure 3.8. If there is an application which needs  $k$  simultaneous connections from  $A$  to  $D$  and all the  $k$  connections must follow the same route, we can obtain the blocking probability by MFT. The MFT is shown at Figure 3.9, and the blocking probability is  $1 - P_S(t)$ . Let  $c$  for all links be 10, and we calculate the blocking probability.



**Figure 3.9:** The MFT of example 3.2.3

**SHARPE File** — ms/app

format 8

epsilon results 0.000000000001

bind

lambda 0.1

mu 0.1

t 20000

end

markov link readprobs

10 9 lambda

9 8 lambda

8 7 lambda

7 6 lambda

6 5 lambda

5 4 lambda

4 3 lambda

3 2 lambda

2 1 lambda

1 0 lambda

0 1 10 \* mu

1 2 9 \* mu

2 3 8 \* mu

3 4 7 \* mu

4 5 6 \* mu

5 6 5 \* mu

6 7 4 \* mu

7 8 3 \* mu

8 9 2 \* mu

9 10 mu

end



10 1.0

end

\*debug mstree

mstree net(t)

basic link1:0 prob(value(t; link, 0))

basic link1:1 prob(value(t; link, 1))

basic link1:2 prob(value(t; link, 2))

basic link1:3 prob(value(t; link, 3))

basic link1:4 prob(value(t; link, 4))

basic link1:5 prob(value(t; link, 5))

basic link1:6 prob(value(t; link, 6))

basic link1:7 prob(value(t; link, 7))

basic link1:8 prob(value(t; link, 8))

basic link1:9 prob(value(t; link, 9))

basic link1:10 prob(value(t; link, 10))

basic link2:0 prob(value(t; link, 0))

basic link2:1 prob(value(t; link, 1))

basic link2:2 prob(value(t; link, 2))

basic link2:3 prob(value(t; link, 3))

basic link2:4 prob(value(t; link, 4))

basic link2:5 prob(value(t; link, 5))

basic link2:6 prob(value(t; link, 6))

basic link2:7 prob(value(t; link, 7))

basic link2:8 prob(value(t; link, 8))

basic link2:9 prob(value(t; link, 9))

basic link2:10 prob(value(t; link, 10))

basic link3:0 prob(value(t; link, 0))

basic link3:1 prob(value(t; link, 1))

basic link3:2 prob(value(t; link, 2))

basic link3:3 prob(value(t; link, 3))  
 basic link3:4 prob(value(t; link, 4))  
 basic link3:5 prob(value(t; link, 5))  
 basic link3:6 prob(value(t; link, 6))  
 basic link3:7 prob(value(t; link, 7))  
 basic link3:8 prob(value(t; link, 8))  
 basic link3:9 prob(value(t; link, 9))  
 basic link3:10 prob(value(t; link, 10))  
 basic link4:0 prob(value(t; link, 0))  
 basic link4:1 prob(value(t; link, 1))  
 basic link4:2 prob(value(t; link, 2))  
 basic link4:3 prob(value(t; link, 3))  
 basic link4:4 prob(value(t; link, 4))  
 basic link4:5 prob(value(t; link, 5))  
 basic link4:6 prob(value(t; link, 6))  
 basic link4:7 prob(value(t; link, 7))  
 basic link4:8 prob(value(t; link, 8))  
 basic link4:9 prob(value(t; link, 9))  
 basic link4:10 prob(value(t; link, 10))  
 basic link5:0 prob(value(t; link, 0))  
 basic link5:1 prob(value(t; link, 1))  
 basic link5:2 prob(value(t; link, 2))  
 basic link5:3 prob(value(t; link, 3))  
 basic link5:4 prob(value(t; link, 4))  
 basic link5:5 prob(value(t; link, 5))  
 basic link5:6 prob(value(t; link, 6))  
 basic link5:7 prob(value(t; link, 7))  
 basic link5:8 prob(value(t; link, 8))  
 basic link5:9 prob(value(t; link, 9))  
 basic link5:10 prob(value(t; link, 10))  
 or slink1 link1:3 link1:4 link1:5 link1:6 link1:7 link1:8 link1:9 link1:10  
 or slink2 link2:3 link2:4 link2:5 link2:6 link2:7 link2:8 link2:9 link2:10

```

or slink3 link3:3 link3:4 link3:5 link3:6 link3:7 link3:8 link3:9 link3:10
or slink4 link4:3 link4:4 link4:5 link4:6 link4:7 link4:8 link4:9 link4:10
or slink5 link5:3 link5:4 link5:5 link5:6 link5:7 link5:8 link5:9 link5:10
and and4l slink5 slink3
and and4r slink2 slink3
or or3l slink2 and4l
or or3r slink5 and4r
and and2l slink1 or3l
and and2r slink4 or3r
or top:1 and2l and2r
end

```

```

loop t, 5, 100, 5
expr 1-sysprob(net, top:1; t)
expr 1-value(t;link,10)-value(t;link,9)
expr 1-value(t;link,10)-value(t;link,9)-value(t;link,8)-value(t;link,7)
bind temp 1-value(t;link,10)-value(t;link,9)-value(t;link,8)-value(t;link,7)
expr temp-value(t;link,6)-value(t;link,5)
expr temp-value(t;link,6)-value(t;link,5)-value(t;link,4)-value(t;link,3)
end

end

```

**Result** Transient analysis of the application at  $\lambda = 0.1$

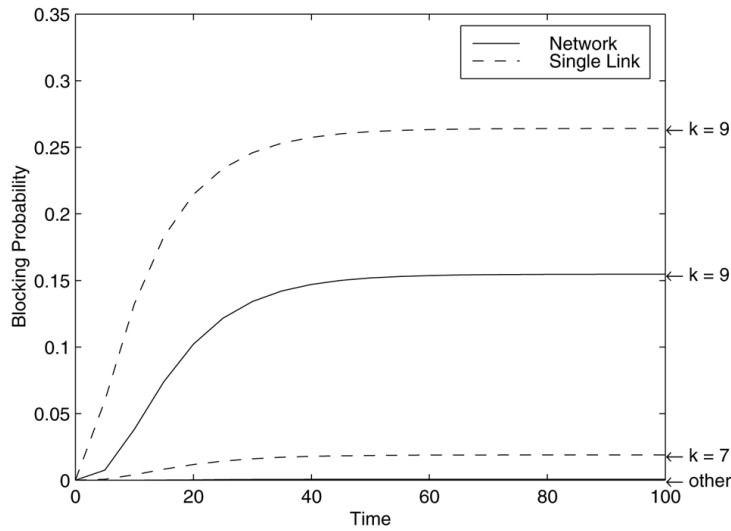


Figure 3.10: Transient analysis

## 3.3 Markov Regenerative Process [17]

### 3.3.1 Specification of model

```

mrgp name {(param_list)}
* section 1: transitions and transition distributions
< nodename1 edgetype nodename2 ep>
* section 2: rewards (optional)
{reward
< name expression>}
end

```

where *nodename1* is the starting node and *nodename2* is the destination node as in Markov and semi-Markov models, *edgetype* is either for Markov regenerative edges, or for non-regenerative edges, *ep* represents a distribution function, which could be **zero**, **inf**, **prob**(*p*),

**exp**( $\lambda$ ), **gen**, **cgen**, **tgen**, **cdf**, **Erlang** ( $n, \lambda$ ), **hypoexp** ( $\mu_1, \mu_2$ ), **hyperexp** ( $\mu_1, p_1, \mu_2, p_2$ ), **mixture** ( $p_1, p_2, \mu$ ), **defective** ( $p, \mu$ ), **inst\_unavail** ( $\lambda, \mu$ ), **ss\_unavail** ( $\lambda, \mu$ ), **oneshot** ( $p$ ), **activeE** ( $\mu$ ), **activeU** ( $\mu_1, \mu_2$ ), **standbyE** ( $\mu, \mu_{sense}$ ), **standbyU** ( $\mu_1, \mu_2, \mu_{sense}$ ), **binomial** ( $\lambda, k, n$ ), **kofn\_ftree** ( $\lambda, k, n$ ), **kofn\_block** ( $\lambda, k, n$ ), or any of user-defined distribution functions. Detailed description of the first 8 distribution functions can be found in Appendix B of [14].

### 3.3.2 System analysis functions

Only steady-state solution of MRGP models is given and the following functions are supported:

- **prob** (*sys\_name*, *nodename* {; *arglist*})

Gets the steady state probability for node *nodename* of the MRGP model named *sys\_name*.

- **exrss** (*sys\_name*{; *arglist*})

Calculates the expected steady-state reward rate value.

### 3.3.3 Example – Cellular Networks with Generally Distributed Hand-off Traffic

#### Source

S. Dharmaraja, and K. Trivedi, Performance Analysis of Cellular Networks with Generally Distributed Hand-off Traffic, COMMUNICATED, 2001.

## Description

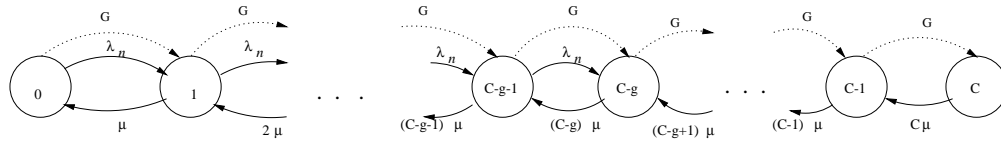
Consider a single cell in a TDMA (Time Division Multiple Access) wireless system, where the base transceiver system of the cell has  $N$  base repeaters, one controller and a local area network connecting these subsystems. Each base repeater provides  $M$  time-division-multiplexed channels. The cell reserves one channel for signaling transfer (namely control channel), which resides in one of  $N$  base repeaters. Therefore, the total number of available channels for calls in the cell is  $NM - 1 (= C)$ . For convenience in demonstrating the approach, we assume that the system has hexagonal geometry and that the cellular system is homogeneous. That is, all the cells are identical and have the same statistical behavior.

A call is accepted only when the cell can find a channel not in use, otherwise, the call is rejected. Call arrivals in cellular system can be classified as new calls and hand-off calls. New calls are generated by mobile originating or mobile terminating connections established in the initial cells, whereas hand-off calls are ongoing calls transferring from other cells. A hand-off call could fail due to insufficient bandwidth available in the new cell, and in such case, a drop of hand-off call occurs.

The dropping of a hand-off call is considered more severe than the blocking of a new call. One method ([7, 9]) to reduce the dropping probability of hand-off calls is to reserve a fixed number of channels exclusively for hand-off calls. These exclusively reserved channels are referred as *guard* channels. For example, if the total number of channels is  $C$  and the number of guard channels in the channel pool is  $g$ , then the number of available channels for new calls is  $C - g$ .

We assume that an ongoing call (new or hand-off) completion times are exponential with parameter  $\mu_d$  and the time at which the mobile station engaged in the call departs the cell are exponential with parameter  $\mu_h$ . We also assume that the inter-arrival times

of hand-off calls are generally distributed with distribution function  $G(t)$  and with finite mean  $1/\lambda_h$  which is independent of new calls arrival time. Note that new calls who find all  $C - g$  channels are busy leave the system whereas hand-off calls who find all  $C$  channels are busy leave the system. The state transition diagram for this model is shown in Figure 3.11.



**Figure 3.11:** State transition diagram using MRGP modeling

**SHARPE File** — *mrqp/cellular*

format 8

bind

lambdaE 63

lambda 49

mu 1

end

\* C = 5, g = 3

mrqp cellular5\_3

0 - 1 exp(lambda)

1 - 0 exp(mu)

1 - 2 exp(lambda)

2 - 1 exp(2\*mu)

3 - 2 exp(3\*mu)

4 - 3 exp(4\*mu)

5 - 4 exp(5\*mu)

0 @ 1 Erlang(3, lambdaE)

```

1 @ 2 Erlang(3, lambdaE)
2 @ 3 Erlang(3, lambdaE)
3 @ 4 Erlang(3, lambdaE)
4 @ 5 Erlang(3, lambdaE)
reward
2 1
3 1
4 1
5 1
end

```

```
* C = 6, g = 3
```

```

mrgp cellular6_3
0 - 1 exp(lambda)
1 - 0 exp(mu)
1 - 2 exp(lambda)
2 - 1 exp(2*mu)
2 - 3 exp(lambda)
3 - 2 exp(3*mu)
4 - 3 exp(4*mu)
5 - 4 exp(5*mu)
6 - 5 exp(6*mu)
0 @ 1 Erlang(3, lambdaE)
1 @ 2 Erlang(3, lambdaE)
2 @ 3 Erlang(3, lambdaE)
3 @ 4 Erlang(3, lambdaE)
4 @ 5 Erlang(3, lambdaE)
5 @ 6 Erlang(3, lambdaE)
reward
3 1
4 1

```



```

5    1
6    1
end

* C = 7, g = 3

mrgp  cellular7_3
0 - 1  exp(lambda)
1 - 0  exp(mu)
1 - 2  exp(lambda)
2 - 1  exp(2*mu)
2 - 3  exp(lambda)
3 - 2  exp(3*mu)
3 - 4  exp(lambda)
4 - 3  exp(4*mu)
5 - 4  exp(5*mu)
6 - 5  exp(6*mu)
7 - 6  exp(7*mu)
0 @ 1  Erlang(3, lambdaE)
1 @ 2  Erlang(3, lambdaE)
2 @ 3  Erlang(3, lambdaE)
3 @ 4  Erlang(3, lambdaE)
4 @ 5  Erlang(3, lambdaE)
5 @ 6  Erlang(3, lambdaE)
6 @ 7  Erlang(3, lambdaE)
reward
4    1
5    1
6    1
7    1
end

```

```
expr prob(cellular5_3, 5)
expr exrss(cellular5_3)
expr prob(cellular6_3, 6)
expr exrss(cellular6_3)
expr prob(cellular7_3, 7)
expr exrss(cellular7_3)

end
```

## 3.4 Reliability Block Diagrams

### 3.4.1 Specification of model [14]

A reliability block diagram is specified by:

```
block name { (param_list) }
<blockline>
end
```

An *blockline* has one of the following forms:

1. **comp** *name ep*

This is a basic component type. It is assigned a name, and an exponential polynomial.

2. **parallel** *name name name* { *name ...* }

This represents components combined in parallel. The parallel system is assigned the first name, and is composed of the rest of the names. There must be at least two components.

3. **or** *name name name { name ... }*

This represents components combined in series. The series system is assigned the first name, and is composed of the rest of the names. There must be at least two components.

4. **kofn** *name expression, expression, name*

This represents a  $k$ -out-of- $n$  system having identical components. The gate is assigned the first name. The first expression gives  $k$  and the second expression gives  $n$ ; the second name gives a component or sub-block. The first name is assumed to consist of  $n$  identically distributed (independent) copies of the second name. In order for the system to be operating,  $k$  of the components must be operating.

5. **kofn** *name expression, expression, name name { name ... }*

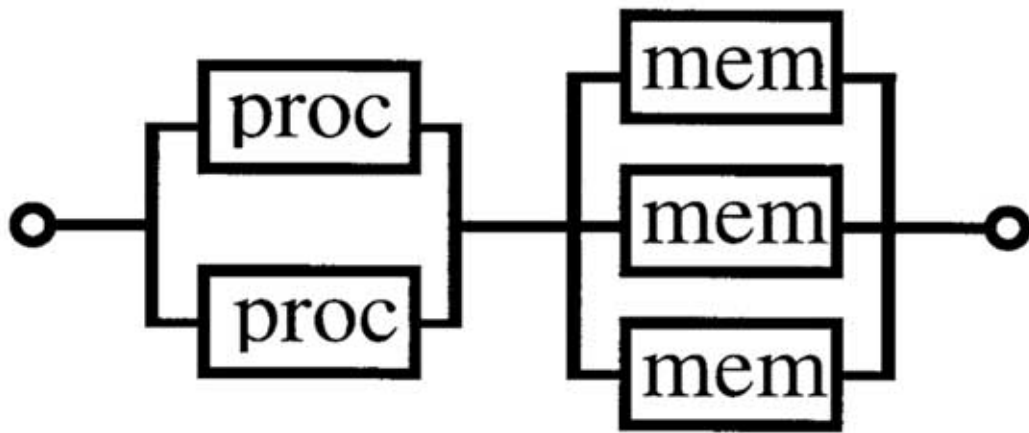
This represents a  $k$ -out-of- $n$  system whose components need not be identical. The system is assigned the first name. The first expression gives  $k$  and the second expression gives  $n$ . The names following the second expression are the components to the system; there must be at least two.

Detailed description of how to analyze reliability block diagrams can be found in Appendix B of [14].

### 3.4.2 Example – 2 Processors, 3 Memories System

#### Description

A system has 2 processors and 3 Memories. Each processor has a failure rate  $\lambda_p$ . Each memory has a failure rate  $\lambda_m$ . The system is up if at least one processor and at least  $k$  (1 or 2) memories are up. The reliability block diagram for  $k = 1$  is shown in Figure 3.12.



**Figure 3.12:** Reliability block diagram for the 2 processors, 3 memories system

**SHARPE File** — *block/2p3m.block*

- \* Two-processors, three-memories system
- \* Use a block diagram to model system reliability
- \* k is the minimum number of memories needed

format 8

```

block nodep(k)
comp proc exp(lambdap)
comp mem exp(lambdam)
parallel procs proc proc
kofn mems k,3,mem
series top procs mems
end

```

- \* Now assign failure rate values

```

bind
lambdap 1/720
lambdam 1/(2*720)
end

* Compare mean time to system failure under
* two conditions: a minimum of
* one memory required vs. 2 memories
* find the difference between the use of tvalue and value
expr mean(nodep;1), mean(nodep;2), mean(nodep;1)/mean(nodep;2)

* Now compare system unreliabilities
func unrel1(t) tvalue(t;nodep;1)
func unrel2(t) tvalue(t;nodep;2)
loop t,0,50,10
expr unrel1(t), unrel2(t)
end

end

```

## 3.5 Fault Trees

### 3.5.1 Specification of model

A fault tree is specified by the following:

```

ftree name { ( param_list ) }
<ftreeline>
end

```

An *ftreeline* has one of the following forms:

1. **basic** *name ep*

This is a basic component type. It is assigned a name, and an exponential polynomial. Whenever this name appears later in the fault tree specification, it is interpreted as being a physically distinct copy of an event type having the assigned exponential polynomial.

2. **repeat** *name ep*

This is also a basic event assigned a name and an exponential polynomial. In this case, whenever this name appears later in the fault tree specification, it is interpreted as being the same physical event.

3. **not** *name name*

This represents a “not” gate. The gate output is assigned the first name, and the second names form the input to the gate. See the example C.1.2.

4. **transfer** *name name*

The second name must have been previously defined using **basic** or **repeat**. Whenever the first name appears later in the fault tree specification, it is interpreted as being the same physical component as the second name.

5. **and** *name name name { name ... }*

This represents an “and” gate. The gate is assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs.

6. **nand** *name name name { name ... }*

This represents a “nand” gate. The gate output is assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs. See the example C.1.1.

7. **or** *name name name { name ... }*

This represents an “or” gate. The gate is assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs.

8. **nor** *name name name { name ... }*

This represents a “nor” gate. The gate is output assigned the first name, and the rest of the names form the inputs to the gate. There must be at least two inputs. See the example C.1.1.

9. **kofn** *name expression, expression, name*

This represents a *k*-out-of-*n* gate having identical inputs. The gate is assigned the first name. The first expression gives *k* and the second expression gives *n*. The inputs to the gate are assumed to be *n* identically distributed, independent copies of the second name.

10. **nkofn** *name expression, expression, name*

This represents a *not k*-out-of-*n* gate having identical inputs. The gate output is assigned the first name. The first expression gives *k* and the second expression gives *n*. The inputs to the gate are assumed to be *n* identically distributed, independent copies of the second name.

11. **kofn** *name expression, expression, name name { name ... }*

This represents a *k*-out-of-*n* gate whose inputs need not be identical. The gate is assigned the first name. The first expression gives *k* and the second expression gives *n*. The names following the second expression are the inputs to the gate; there must be at least two.

12. **nkofn** *name expression, expression, name name { name ... }*

This represents a *not k*-out-of-*n* gate whose inputs need not be identical. The gate is assigned the first name. The first expression gives *k* and the second expression gives

$n$ . The names following the second expression are the inputs to the gate; there must be at least two. The inputs are assumed to be configured so that the system only fails if  $k$  of the inputs fail. See the example C.1.2.

In forms 2 through 8, the names making up the block must already be defined.

### 3.5.2 System analysis functions

New analysis functions and new features are listed as the following. Other analysis functions are described in Appendix B of [14].

1. **mincuts**(*system\_name* {; *arglist*})

This prints out the set of mincuts of a fault tree (See the example C.1.3).

2. Results for gate:

User can obtain results at each gate output by assigning the name of the gate to *state\_eword* in corresponding function. For example, if the **cdf** is asked for gate, *gn*, in a fault tree, *ft*, **cdf**(*ft*, *gn*) can give the result.

3. Importance measure for an event:

Three types of importance measure can be obtained from a fault tree model (see the example C.1.4):

(a) **bimpt**(*t*; *system\_name*, *event\_name* {; *arglist*})

This gives Birnbaum's importance for event, *event\_name*, at time *t*.

(b) **cimpt**(*t*; *system\_name*, *event\_name* {; *arglist*})

This gives criticality importance for event, *event\_name*, at time *t*.

(c) **simplt**(*system\_name*, *event\_name* {; *arglist*})

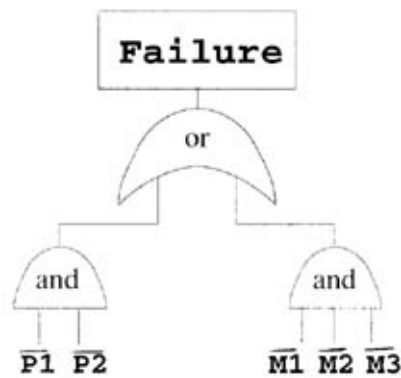
This gives structural importance for event, *event\_name*.



### 3.5.3 Example – 2 Processors, 3 Memories System

#### Description

This is the same system introduced in chapter 3.4.2. The corresponding fault tree is in Figure 3.13, where  $P1$  and  $P2$  represent the two processors, and  $M1$ ,  $M2$ , and  $M3$  denote the three memories, respectively. Furthermore,  $\mu_p$  and  $\mu_m$  have been introduced as independent repair rates for each processor and each memory, respectively. Then, the instantaneous unavailability of the system has been calculated via the model named *indrep* in the SHARPE file listed at the chapter 3.5.3.



**Figure 3.13:** Fault tree for the 2 processors, 3 memories system

**SHARPE File** — *ftree/2p3m.ftree*

\* 2 processors, 3 memories system modeled by fault tree

format 8

```

ftree nodepf(k)
basic proc exp(lambdap)
basic mem exp(lambdam)
and procs proc proc
kofn mems (4-k),3,mem
or top procs mems
end

```

\* Now assign failure rate values

```

bind
lambdap 1/720
lambdam 1/(2*720)
end

```

\* note the difference in kofn of ftree with block

\* Compare answers obtained by two

\* distinct models of the same system

```

expr mean(nodepf;1), mean(nodepf;2), mean(nodepf;1)/mean(nodepf;2)

```

\* Assume Independent Failure And Independent Repair

\* model system insta. availability

```

ftree indrep(k)
basic proc inst_unavail(lambdap,mup)
basic mem inst_unavail(lambdam,mum)
and procs proc proc
kofn mems (4-k),3,mem
or top procs mems
end

```

\* Assign Repair Rate Values

```

bind
mup 1/2.5

```

```

mum 1/2.5
end
* Now compare system unavailabilities
func unavail1(t) tvalue(t;indrep;1)
func unavail2(t) tvalue(t;indrep;2)
loop t,0,50,10
  expr unavail1(t), unavail2(t)
end

end

```

## 3.6 Reliability Graphs

### 3.6.1 Specification of model

A reliability graph is specified by the following:

```

relgraph name { ( param_list ) }
* section 1: unidirectional edges
<edge_name edge_name ep { transfer edge1_name edge1_name { edge2_name
edge2_name ... } } >
* section 2: bidirectional edges (optional)
{ bidirect
<edge_name edge_name ep { transfer edge1_name edge1_name { edge2_name
edge2_name ... } } > }
end

```

The **transfer** part in the above specification is the extension that defines the repeated edges. The *edge1* from the first *edge1\_name* to the second *edge1\_name* is repeated for the

*edge* from the first *edge\_name* to the second *edge\_name*. So are the optional edges from the first *edge\_name* to the second *edge\_name*. Examples of repeated edges are listed in chapter 3.6.3.

### 3.6.2 System analysis functions

Two new types of system analysis functions are integrated as the following (for others, see Appendix B of [14]):

1. Mincuts and minpaths set:

(a) **mincuts**(*system\_name* {; *arglist*})

This prints out the set of mincuts of a reliability graph. See the example C.2.1.

(b) **minpaths**(*system\_name* {; *arglist*})

This prints out the set of minpaths of a reliability graph. See the example C.2.2.

2. Importance measure for an edge:

Three types of importance measure can be obtained from a reliability graph model (see the example C.2.3):

(a) **bimpt**(*t*; *system\_name*, *node\_name*, *node\_name* {; *arglist*})

This gives Birnbaum's importance for edge, (*node\_name*, *node\_name*), at time *t*.

(b) **cimpt**(*t*; *system\_name*, *node\_name*, *node\_name* {; *arglist*})

This gives criticality importance for edge, (*node\_name*, *node\_name*), at time *t*.

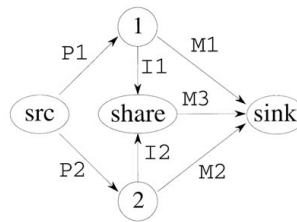
(c) **simpt**(*system\_name*, *node\_name*, *node\_name* {; *arglist*})

This gives structural importance for edge, (*node\_name*, *node\_name*).

### 3.6.3 Examples

#### 2 Processors, 3 Memories System with Inter-connection Dependence

**Description** This is still a system with 2 processors and 3 memories. Compared to the system mentioned in chapter 3.4.2 and chapter 3.5.3, inter-connection dependence has been considered. Processor  $P1$  only uses memory  $M1$  and  $M3$ , and processor  $P2$  only uses memory  $M2$  and  $M3$ . The system is up when at least one processor and one memory are working. In the following SHARPE file, the model *rel\_proc\_mem2* is based on repeated edges. The reliability graph for the model *rel\_proc\_mem* is shown in Figure 3.14.



**Figure 3.14:** Reliability graph for the 2 processors, 3 memories system with inter-connection dependence without repeated edges

**SHARPE File** — *relgraph/repeat.txt*

\* reliability graph for  
\* 2—processor,  
\* 3—memory system

```
relgraph rel_proc_mem
src P1 exp(1/Ptime)
src P2 exp(1/Ptime)
P1 sink exp(1/Mtime)
P2 sink exp(1/Mtime)
```

```

P1 share inf
P2 share inf
share sink exp(1/Mtime)
end

bdd on

relgraph rel_proc_mem2
src P1 exp(1/Ptime)
src P2 exp(1/Ptime)
P1 sink exp(1/Mtime)
P2 sink exp(1/Mtime)
P1 sink exp(1/Mtime) transfer P2 sink
end

bind
Ptime 720
Mtime 2*720
end

pqcdf(rel_proc_mem)
cdf(rel_proc_mem)

pqcdf(rel_proc_mem2)
cdf(rel_proc_mem2)

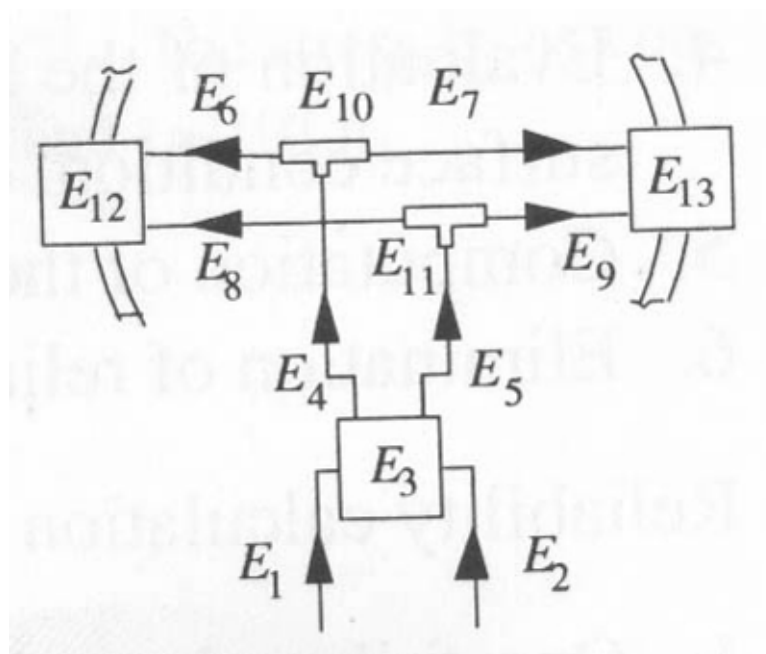
end

```

## **An Electrical-pyrotechnic System**

**Source** A. Birolini, *Quality and Reliability of Technical Systems*, Springer-Verlag, Berlin Heidelberg, New York, 1994.

**Description** To separate a satellite's protective shielding, a special electrical-pyrotechnic system shown in Figure 3.15 is used. An electrical signal comes through the cables  $E_1$  and  $E_2$  (redundancy) to the electrical-pyrotechnic signal to explosive charges for guillotining bolts  $E_{12}$  and  $E_{13}$  of the tensioning belt. The charges can be ignited from two sides, although one ignition will suffice (redundancy). For fulfillment of the required function, both bolts must be exploded simultaneously. Calculate the probability of failure of this separation system.



**Figure 3.15:** A special electrical-pyrotechnic system

**SHARPE File** — *relgraph/ex2.15*

relgraph ex2.15(e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13)

src p1 exp(e1)

src p1 exp(e2)

p1 p2 exp(e3)

```

p2 p4 exp(e4) transfer p8 p10
p2 p3 exp(e5) transfer p8 p9
p6 p7 exp(e6)
p12 p13 exp(e7)
p5 p7 exp(e8)
p11 p13 exp(e9)
p4 p6 exp(e10) transfer p10 p12
p3 p5 exp(e11) transfer p9 p11
p7 p8 exp(e12)
p12 sink exp(e13)
end

```

```

pqcdf(ex2.15; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13)

```

```

end

```

## 3.7 Series-parallel Acyclic Directed Graphs

### 3.7.1 Specification of model

A series-parallel graph is specified as follows:

```

graph name {(param_list)}
<name { name } >
end
<graphline>
end

```

A *graphline* has one of the following forms:



1. **dist** *name ep*

this assigns the given *ep*, which is a defined distribution function, to the given graph node. An *ep* must be specified for each graph node.

2. **exit** *name exit\_type*

This assigns the given exit type to the given node. For every node that has more than one exiting edge, an exit type must be specified. If a graph called *g* has more than one entrance node (node with no predecessors), then SHARPE supplies a dummy entrance node called *E.g* with zero exponential polynomial and edges leading from *E.g* to each user-specified entrance node. When this is the case, the user must supply an exit type for the node *E.g*.

3. **prob** *name name expression*

The expression gives a probability value to be assigned to the edge going from the first name to the second name. For each node *x* that has *n* successors and whose exit type is **prob**, probability values must be assigned to at least  $n - 1$  of the edges leading out of *x*. If values are given for all of the edges, the sum of the values must be 1. If one value is missing, the sum of the values must be less than 1 and SHARPE will compute the missing value.

4. **multipath**

This line requests multiple-path information for the system. Whenever there are probabilistic subgraphs that are not inside maximum, minimum, or *k*-out-of-*n* subgraphs, SHARPE considers the graph to contain more than one path. If multiple-path information is requested, SHARPE will compute for each path the probability of taking the path and the conditional distribution for the time-to-finish, given that the path is taken.

The exit types (*exit\_type*) are

1. **prob**

The parallel subgraphs are probabilistic.

2. **max**

All of the parallel subgraphs must complete before going on.

3. **min**

One of the parallel subgraphs must complete before going on.

4. **kofn** *expression, expression*

The first expression gives  $k$  and the second expression gives  $n$ ;  $k$  out of the  $n$  parallel subgraphs must complete before going on. If this exit type is specified for a graph with exactly one successor node, that node is assumed to be duplicated  $n$  times, with each copy being identically distributed. Except for this case, it is required that a node with **kofn** exit type have exactly  $n$  following parallel subgraphs.

Detailed description of how to analyze series-parallel acyclic directed graph can be found in Appendix B of [14].

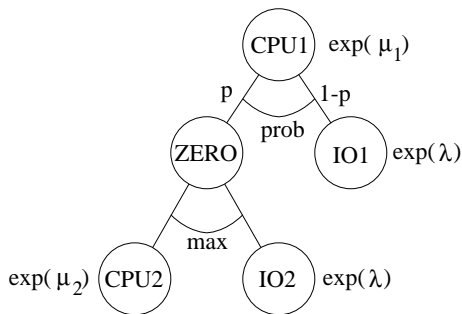
### 3.7.2 Example – A CPU-Input/Output Overlap System

#### Source

D.F. Towsley, J.C. Browne and K.M. Chandy, Models for Parallel Processing within Programs, *CACM*, October, 1978.

## Description

Figure 3.16 shows a series-parallel graph representing one iteration of the program with CPU-Input/Output Overlap. In each iteration of the program, there are two stages. The first stage is always a CPU burst. The second stage consists of either pure I/O, or I/O that may be overlapped with a second CPU burst. As in Figure 3.16, the probability that the second stage contains CPU-I/O overlap is given by  $p$ . In the following SHARPE file, the model *OVERLAP* represents the model in Figure 3.16, while the model *SERIAL* denotes the model without CPU-I/O overlap. The speedup for various values of  $p$  has been computed.



**Figure 3.16:** Precedence graph for the CPU-I/O overlap system

### SHARPE File — *th/24*

\* CPU-I/O overlap

bind

mu1 1 / 0.0376

mu2 1 / 0.125

lambda 1 / 0.14995

end

graph SERIAL(p)

```
cpu1 cpu2
cpu2 io2
cpu1 io1
end
```

```
exit cpu1 prob
prob cpu1 cpu2 p
dist cpu1 exp ( mu1)
dist io1 exp ( lambda)
dist cpu2 exp ( mu2)
dist io2 exp ( lambda)
end
```

```
graph OVERLAP(p)
cpu1 zero1
cpu1 io1
zero1 cpu2
zero1 io2
end
```

```
exit cpu1 prob
prob cpu1 zero1 p
exit zero1 max
dist cpu1 exp ( mu1)
dist zero1 zero
dist io1 exp ( lambda)
dist cpu2 exp ( mu2)
dist io2 exp ( lambda)
end
```

```
expr mean(SERIAL;0.7)
expr mean(OVERLAP;0.7)
```

```
expr mean(SERIAL;0.6)/mean(OVERLAP;0.6)
expr mean(SERIAL;0.7)/mean(OVERLAP;0.7)
expr mean(SERIAL;0.8)/mean(OVERLAP;0.8)
expr mean(SERIAL;0.9)/mean(OVERLAP;0.9)
expr mean(SERIAL;1.0)/mean(OVERLAP;1.0)
```

```
bind
mu1 1 / 0.01
end
```

```
expr mean(SERIAL;1.0)/mean(OVERLAP;1.0)
end
```

## 3.8 Single-chain Product-form Queueing Networks

### 3.8.1 Specification of model

A single-chain product-form queueing network is specified as follows:

```
pfqn name {(param_list)}
* section 1: station-to-station probabilities
<station_name station_name expression>
end
* section 2: station types and parameters
<stationline>
end
section 3: number of customers per chain
<chain_name expression>
end
```

An *blockline* has one of the following forms:

1. *station\_name* **is** *rate*

The station is an infinite server; each job at the server has exponential service-time CDF with the specified rate.

2. *station\_name* **fcfs** *rate*

The station is a first-come-first-serve server. Jobs in the queue are served once at a time; the job being served (if any) has exponential service-time CDF with the specified rate.

3. *station\_name* **ps** *rate*

Jobs at the station share the server. When  $n$  jobs are at the station, each has exponential service-time CDF with rate  $rate/n$ .

4. *station\_name* **lcspr** *rate*

The serving algorithm is "last come first served, preemptive resume".

5. *station\_name* **ms** *number\_of\_servers, rate*

The station contains multiple servers; the number of servers is given by the *expression number\_of\_servers*. Each server has the same rate.

6. *station\_name* **lds** *rate, rate, . . .*

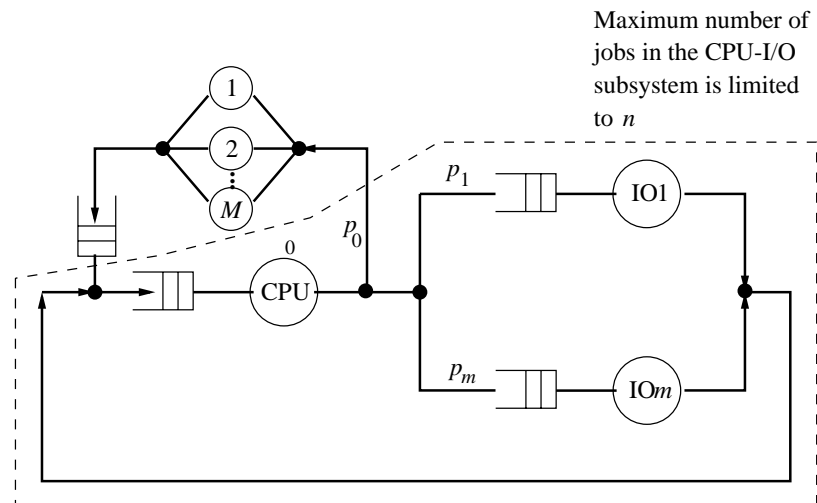
There is one server, whose service rate depends on the number of jobs at the station. The first rate applies when there is one job, the second rate when there are two jobs, and so on. If there are fewer rates given than the maximum number of jobs, the last rate on the line is assigned to all numbers of jobs for which no rate was explicitly given.

Detailed explanation of how to analyze single-chain product-form queueing networks can be found in Appendix B of [14].

### 3.8.2 Example — a Terminal-oriented System with a Limited Number of Memory Partitions [16]

#### Description

This is the example 9.16 in [16]. As shown in Figure 3.17, the system has  $M$  terminals. Only  $n$  active jobs can concurrently share the main memory, which means  $M = n$ . Also, there is an assumption that the main memory is large enough so that no waiting in the job queue is required, which means the station *term* is an infinite server with the key word **is** assigned to it as mentioned in the previous section. The model tested in the following SHARPE file has  $m = 3$ .



**Figure 3.17:** a Terminal-oriented System with a Limited Number of Memory Partitions

**SHARPE File** — *pfqn/9.16-nocon*

- \* This example is Ex 9.16 from the book.
- \* This implements the queueing network ignoring the
- \* memory constraint. This corresponds to  $E[R^*]$  in table 9.12

```
bind
p0 0.05
p1 0.5
p2 0.3
p3 0.15
scpu 89.3
sio1 44.6
sio2 26.8
sio3 13.4
sterm 1/15
end
```

```
pfqn ex9.16(n)
cpu term p0
cpu io1 p1
cpu io2 p2
cpu io3 p3
io1 cpu 1
io2 cpu 1
io3 cpu 1
term cpu 1
end
```

```
cpu fcfs scpu
term is sterm
io1 fcfs sio1
io2 fcfs sio2
io3 fcfs sio3
end
```

```
cust n
end
```



```

func ET(N) scpu*util(ex9.16,cpu;N)*p0
func ER(M) M/ET(M) - 1/sterm
expr ER(10)
expr ER(20)
expr ER(30)
expr ER(40)
expr ER(50)
expr ER(60)
end

```

## 3.9 Multiple-chain Product-form Queueing Networks

### 3.9.1 Specification of model

A multiple-chain product-form queueing network is specified as follows:

```

mpfqn name {(param_list)}
* section 1: station-to-station probabilities for each chain
<chain chain_name
<station_name station_name expression>
end>
end
* section 2: station types and parameters
<stationline>
{ <chain_name expression, ...> }
end>
end

```

\* section 3: number of customers per chain

<*chain\_name expression*>

**end**

Detailed explanation of how to analyze multiple-chain product-form queueing networks can be found in Appendix B of [14].

### **3.9.2 Example — a Terminal-oriented System with a Limited Number of Memory Partitions [16]**

#### **Description**

This is the multiple-chain product-form queueing network version of the system mentioned in chapter 3.8.2.

#### **SHARPE File — *mpfqn/inp9.16b***

\* This example is Ex 9.16 from the book. This implements the queueing

\* network ignoring the

\* memory constraint. This corresponds to  $E[R^*]$  in table 9.12

\* results should be the same as for *pfqn/9.16—nocon*

bind

p0 0.05

p1 0.5

p2 0.3

p3 0.15

scpu 89.3

sio1 44.6

```

sio2 26.8
sio3 13.4
sterm 1/15
end
mpfqm ex9.16(n)
chain cust
cpu term p0
cpu io1 p1
cpu io2 p2
cpu io3 p3
io1 cpu 1
io2 cpu 1
io3 cpu 1
term cpu 1
end
end
cpu fcfs scpu
end
term is sterm
end
io1 fcfs sio1
end
io2 fcfs sio2
end
io3 fcfs sio3
end
end
cust n
end
func ET(N) scpu*mutil(ex9.16,cpu;N)*p0
func ER(M) M/ET(M) - 1/sterm
expr ER(10)

```

```
expr ER(20)
expr ER(30)
expr ER(40)
expr ER(50)
expr ER(60)
end
```

## 3.10 Markov Chains

### 3.10.1 Specification of model

A Markov chain is specified as follows:

```
markov name {(param_list)} { readprobs }
* section 1: transitions and transition distributions
<markov_edgeline>
* section 2: rewards (optional)
{reward { default expression }
<markov_setline>}
end
* section 3: initial state probabilities
{<markov_setline>}
end
{ fastmttf
< name reada >
< name readf >
end }
```

where *markov\_edgeline* are either

*name name expression*

or

**loop** *simple\_var, low, high {,increment }*

*<markov\_edgeline>*

**end**

and *markov\_setline* are either

*name expression*

or

**loop** *simple\_var, low, high {,increment }*

*<markov\_setline>*

**end**

which you can set reward rate or initial values to the node *name*.

Normally, an irreducible Markov chain doesn't have been specified with initial state probabilities, which means it is not necessary for an irreducible Markov chain to have section 3 unless users specify **readprobs**. Also, without initial state probabilities, **tvalue** and **prob** cannot be applied to irreducible Markov chains.

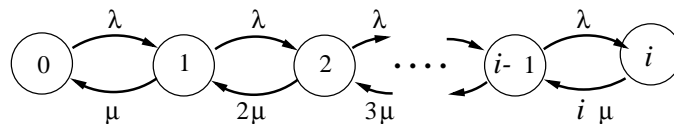
Fast mean time to failure(MTTF) is introduced from the paper [6] and requires the operating system running SHARPE supports IEEE 754 floating point standard. See the example at chapter C.3.1.

Detailed information of how to analyze Markov chains can be found in Appendix B of [14].

### 3.10.2 Example — Erlang Loss Model

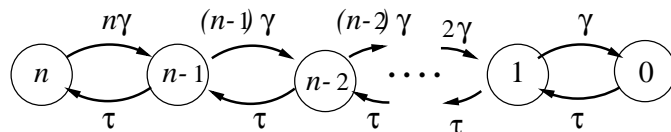
#### Description

Consider a telephone switching system having  $n$  trunks with an infinite caller population. The arrival times are exponentially distributed with rate  $\lambda$  and call holding times are exponentially distributed with average  $\frac{1}{\mu}$ . When an arriving call finds all  $n$  trunks are busy, it is lost without further trying. Given number of non-failed channels, the principal quantity of interest is the *blocking probability*, which is obtained by the steady-state probability that all trunks are busy. The state diagram is shown in Figure 3.18.



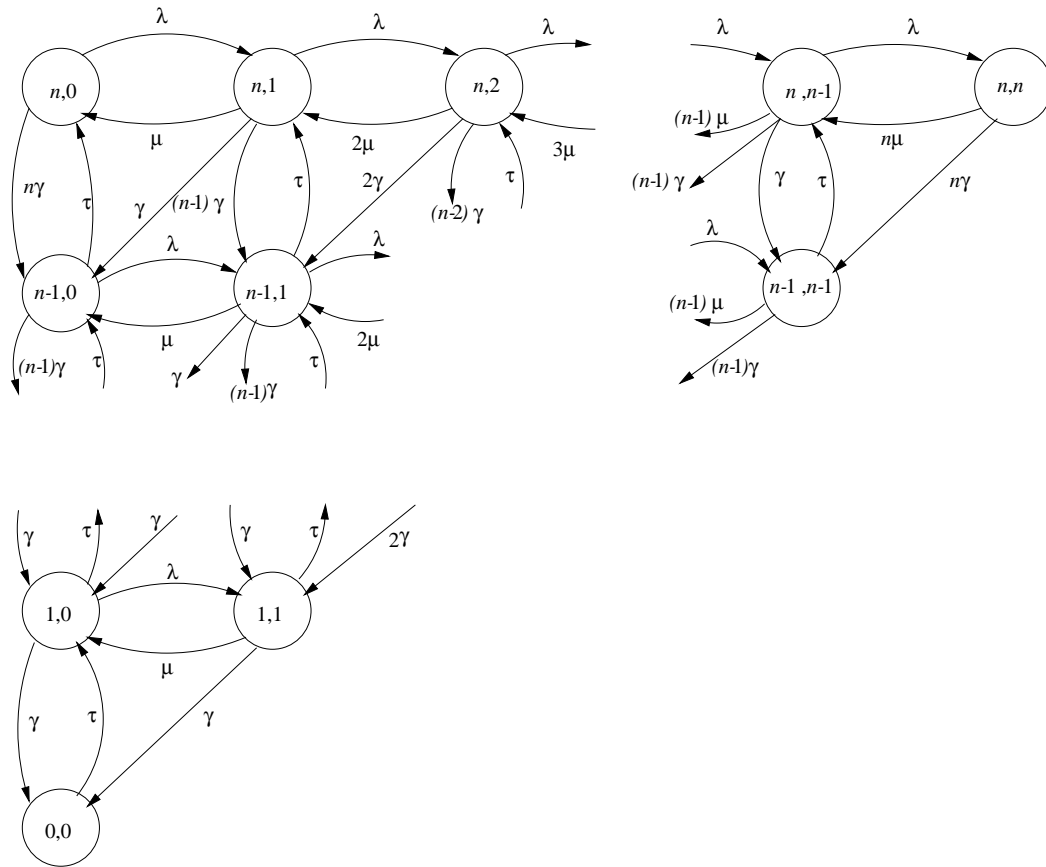
**Figure 3.18:** State diagram for the Erlang loss performance model

Assume that a single repair unit is shared by all the trunks. Also assume that the times to trunks failures and repair are exponentially distributed with rate  $\gamma$  and  $\tau$ , respectively. The availability model is the CTMC in Figure 3.19.



**Figure 3.19:** State diagram for the Erlang loss availability model

The composite model is shown in Figure 3.20. The state  $(i, j)$  represents that  $i$  non-failed trunks and  $j$  calls are currently in the system.



**Figure 3.20:** State diagram for the Erlang loss performability composite model

**SHARPE File** — *bluebook/8.27*

- \* This example is Ex 8.27 from the book.
- \* This implements the Erlang loss model.

format 8

bind

lambda 49

mu 3

MTTF 1000

MTTR 24

end

\* Hierarchical Model

\* Availability submodel

markov perf(C)

loop i,0,C-1

\$(i) \$(i+1) lambda

\$(i+1) \$(i) (i+1)\*mu

end

end

end

\* function to use to define the reward rates for the measure

\* the total call blocking probability

\* Reward function used for  $k > g$

func Rew(C) prob(perf,\$(C);C)

markov hier

loop i,C,1,-1

\$(i) \$(i-1) i/MTTF

\$(i-1) \$(i) 1/MTTR

end

reward

0 1

loop i,1,C

\$(i) Rew(i)

end

end

\* Initial probability



```

$(C) 1
end

loop nb,35,45,1
  bind C nb
  expr exrss(hier)
end

var Td exrss(hier)
loop nb,35,45,1
  bind C nb
  expr Td
end

* Composite model
markov cp
loop j,1,C,1
  * Definition of the Availability part of the model
  * Downwards failure
  loop i,C,j,-1
    $(i)_{$(j-1)} $(i-1)_{$(j-1)} (i-j+1)/MTTF
    $(i-1)_{$(j-1)} $(i)_{$(j-1)} 1/MTTR
    * Definition of the Performance part of the model
    $(i)_{$(j-1)} $(i)_{$(j)} lambda
    $(i)_{$(j)} $(i)_{$(j-1)} j*mu
    * Diagonal failure
    $(i)_{$(j)} $(i-1)_{$(j-1)} (j)/MTTF
  end
end
end
end

```

```

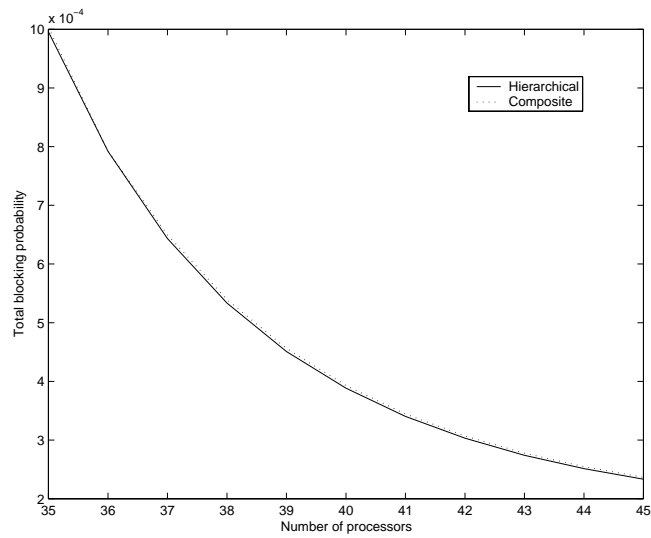
* Outputs
* Total call blocking probability
var Tb sum(i,0,C, prob(cp,$(i) $\$(i)))
var Unavail prob(cp,0 0)

loop nb,35,45,1
  bind C nb
  expr Tb
end

end

```

## Result



**Figure 3.21:** Total blocking probability in the Erlang loss performability model

## 3.11 Semi-Markov Chains

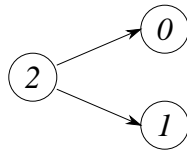
### 3.11.1 Specification of model

A semi-Markov chain is specified as follows:

```
semimark name {(param_List)} { cond | uncond }  
* section 1: transitions and transition distributions  
<nodename1 nodename2 ep>  
* section 2: rewards (optional)  
{reward { default expression }  
<name expression> }  
end  
section 3: initial state probabilities  
{<name expression>}  
end  
{ fastmttf  
< name reada >  
< name readf >  
end }
```

An irreducible semi-Markov chain doesn't have section 3. The key word **fastmttf** is used for fast MTTF [6]. See the example C.3.2.

Detailed information of how to analyze semi-Markov chains can be found in Appendix B of [14].



**Figure 3.22:** A semi-Markov chain

### 3.11.2 Example — Figure 3.22

**SHARPE File** — *semimark/1*

```

semimark main
2 1 gen\
  1, 0, 0\
  -1, 0, -lambda\
  -lambda, 1, -lambda
2 0 exp (.01)
end
end

bind
lambda .02
end

lcdf (main,2)
cdf (main,1)
cdf (main,0)
end
  
```

## 3.12 Generalized Stochastic Petri Nets

### 3.12.1 Specification of model

A generalized stochastic Petri Net(GSPN) is specified as follows:

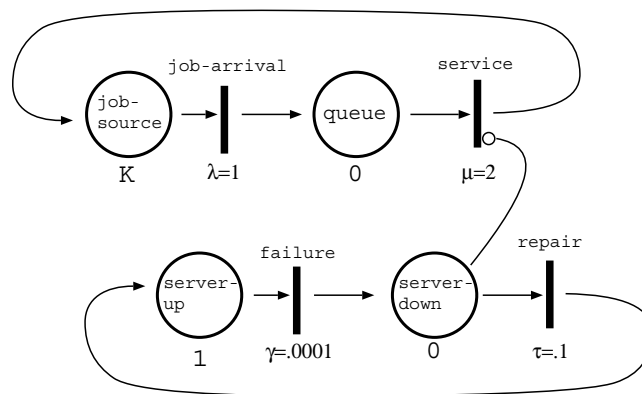
```
gspn name (param_list)  
* section 1: places and initial numbers of tokens  
<place_name expression>  
end  
* section 2: timed transition names, types and rates  
<transition_name ind expression>  
<transition_name dep place_name expression>  
end  
* section 3: immediate transition names, types and weights  
<transition_name ind expression>  
<transition_name dep place_name expression>  
end  
* section 4: place-to-transition arcs and multiplicity  
<place_name transition_name expression>  
end  
* section 5: transition-to-place arcs and multiplicity  
<transition_name place_name expression>  
end  
* section 6: inhibitor arcs and multiplicity  
<place_name transition_name expression>  
end
```

Detailed information of how to analyze generalized Stochastic Petri Nets can be found in Appendix B of [14].

### 3.12.2 Example — M/M/1/K Queue with Server Failure and Repair

#### Description

The system has 1 server with buffer length  $K$ . So  $K$  jobs can be in the system at a time. The exponentially failure and repair rates for the server are  $\gamma$  and  $\tau$ , respectively. See the Figure 3.23.



**Figure 3.23:** GSPN model for queue with server failure and repair

**SHARPE File** — *whitebook/mm1k.gspn*

\* Initialize Variables

bind

LAM 1

MU 2

GAM 0.0001

TAU 0.1

```

inhibtok 1
end

gspn mm1k(K)
* Initial # of Tokens in Places
jobsource K
queue 0
serverup 1
serverdown 0
end
* Rates of Timed Transitions
jobarrival ind LAM
service ind MU
failure ind GAM
repair ind TAU
end
* No Immediate Transitions
end
* Input Arcs
jobsource jobarrival 1
queue service 1
serverup failure 1
serverdown repair 1
end
* Output Arcs
jobarrival queue 1
service jobsource 1
failure serverdown 1
repair serverup 1
end
* Inhibit Arcs
serverdown service inhibtok

```

```
end

var Lreject LAM*prempy(mm1k,jobsource;10)
var Pidle prempy(mm1k,queue;10)
var Preject prempy(mm1k,jobsource;10)
var avqulength etok(mm1k, queue; 10)
var thruput tput(mm1k, service; 10)
var utilization util(mm1k, service; 10)

expr Pidle
expr Lreject, Preject
expr avqulength
expr thruput, utilization
end
```



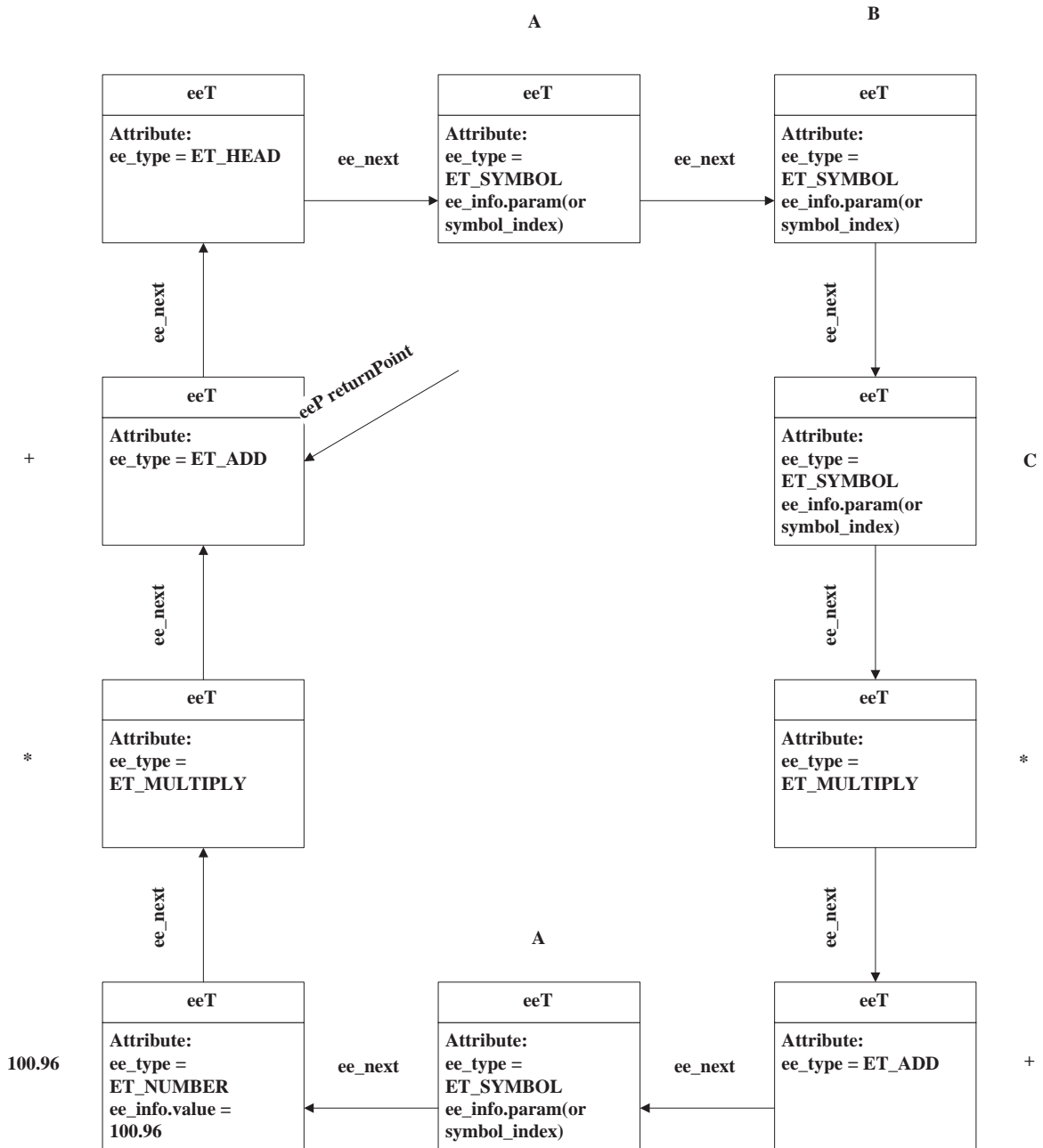
# Appendix A

## SHARPE Data Structure

Important data structures of SHARPE source code are listed here. Rectangles represent instances of data types with the name of each data type at the top of rectangles. These data types are *structures* or *unions* in C language. For the sake of saving space, only important member field(s) are listed at the **attribute** field of each rectangle. Arcs represent pointers. If an arc begins from a rectangle, it is a field of the data type that the rectangle represents. Rectangles are piled together to denote *arrays* in C.

## Basic EXPRESSION Sample

**A + B \* C + A \* 100.96 stored as A B C \* + A 100.96 \* +**



# Advanced Expression I

## Expression List:

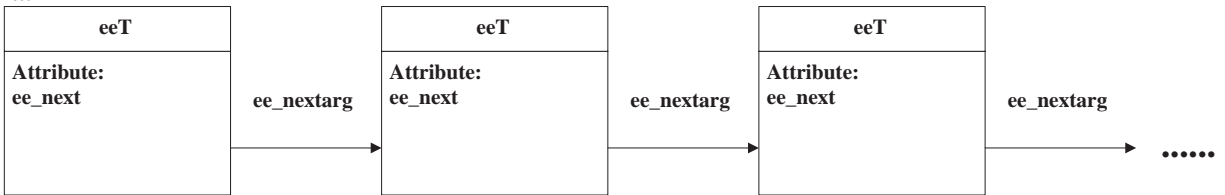
expression, expression, expression, ...

OR

expression

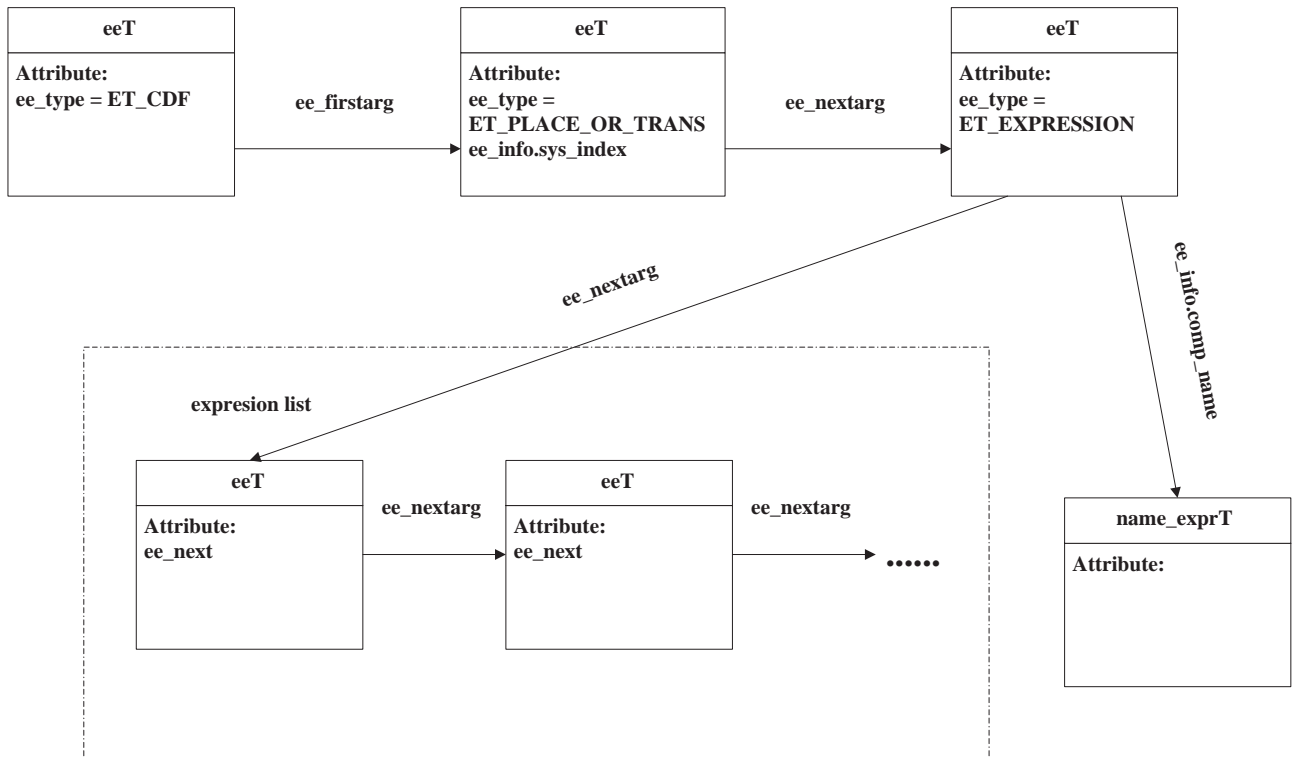
expression

...



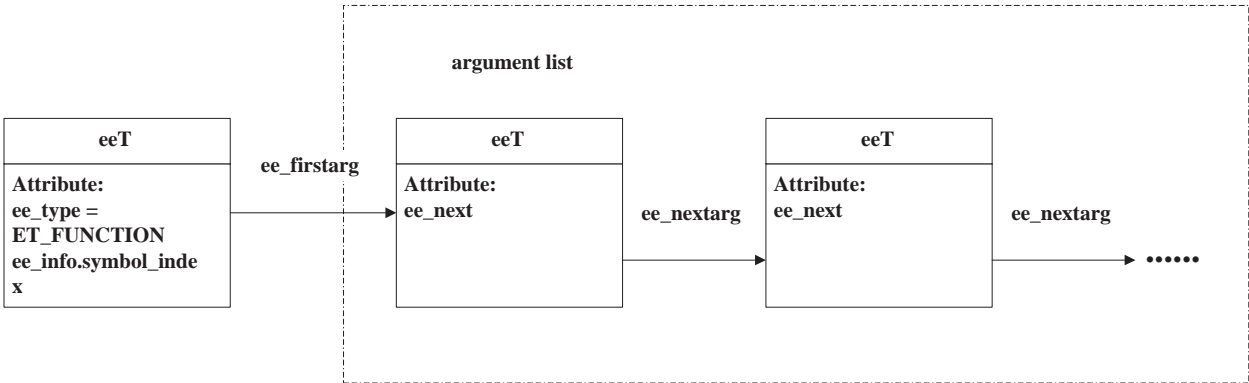
## Buildin Function Node:

CDF(gspn\_g, node1; a, b c)

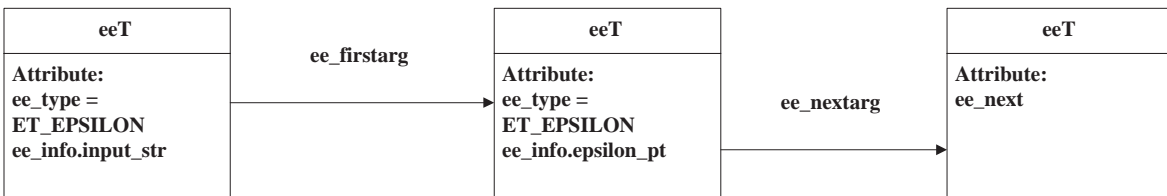


## Advanced Expression II

### User Defined Function



### `epsilon epsilon_id expression`



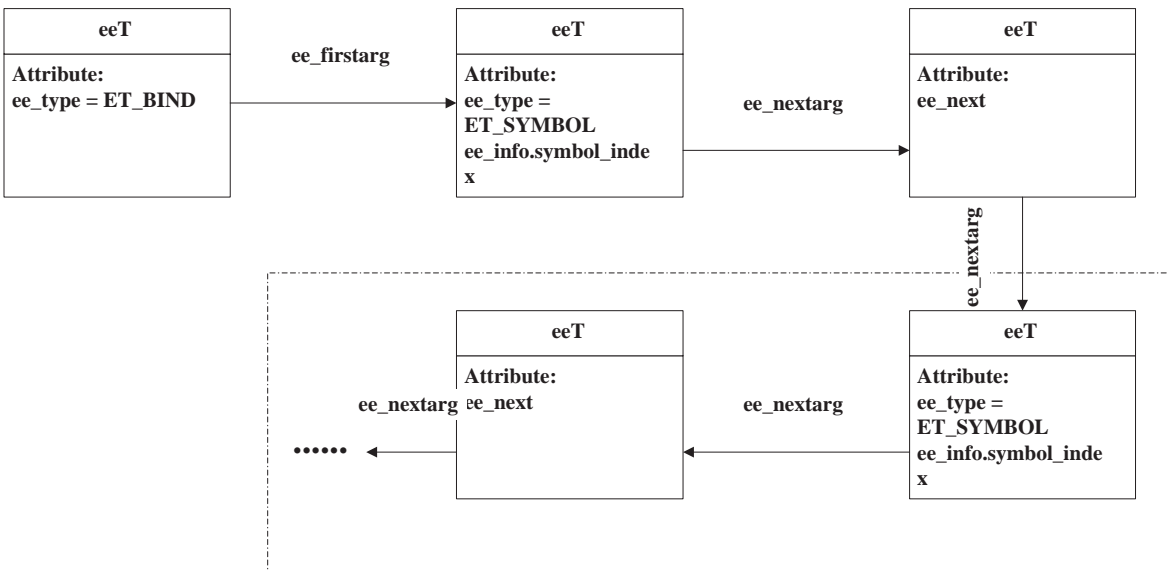
### `bind simple_var expression`

OR

`bind`

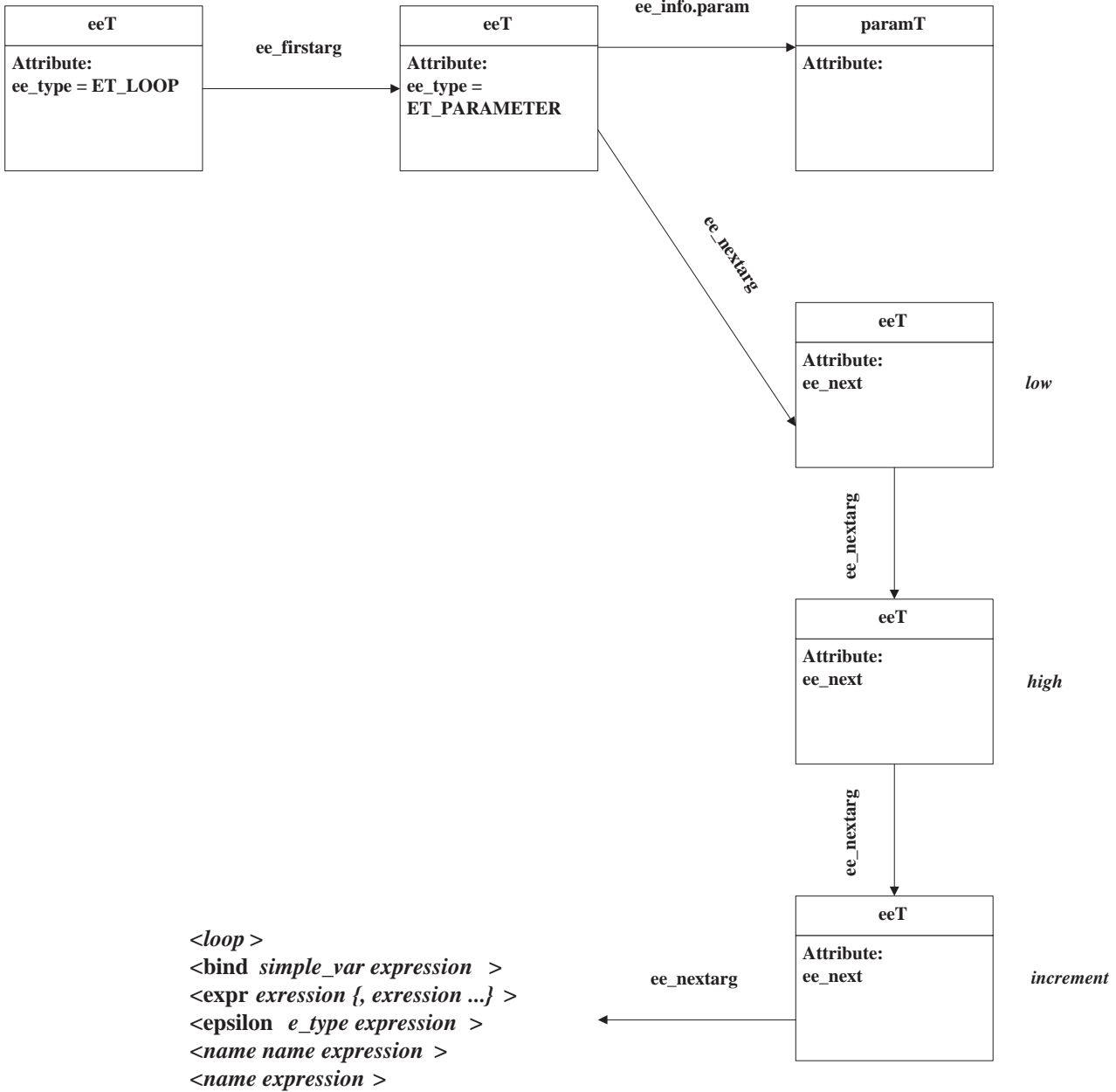
`<simple_var expression >`

`end`



## Advanced Expression III

`loop simple_var, low, high {, increment}`  
`<<loop> | <while_ statement> | <bind simple_var expression> | <expr expression {, expression ...}> | <epsilon e_type expression>`  
`end`

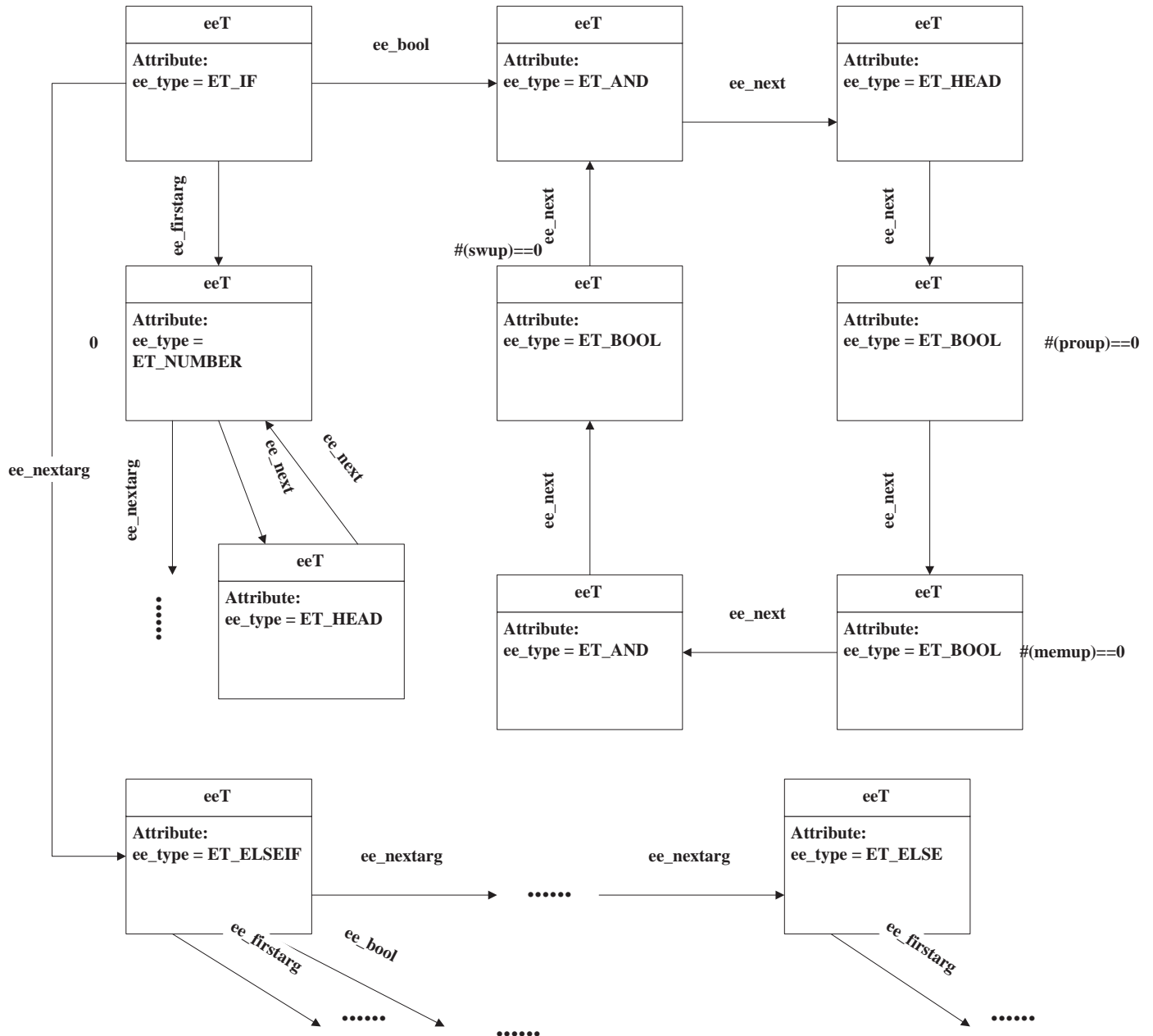


## Advanced Expression IV

### Extended Expression I ---- if- statement

```

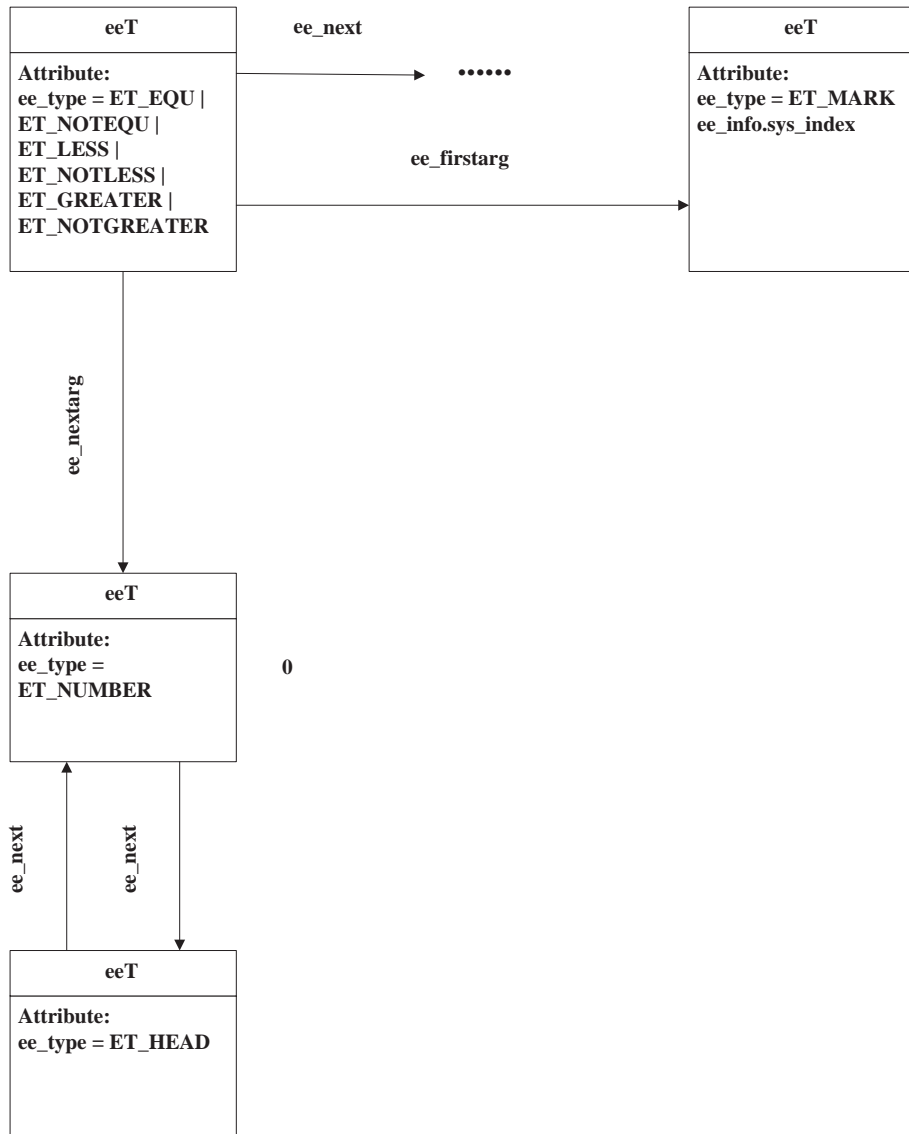
if ((#(procup) == 0) and (#(memup) == 0) and (#(swup) == 0))
0
elseif (.....)
<<if-statement ><bind simple_var expression > | < expression > | <epsilon e_type expression >>
else
<<if-statement ><bind simple_var expression > | < expression > | <epsilon e_type expression >>
end
    
```



# Advanced Expression V

## Extended Expression II

\$(procup)==0

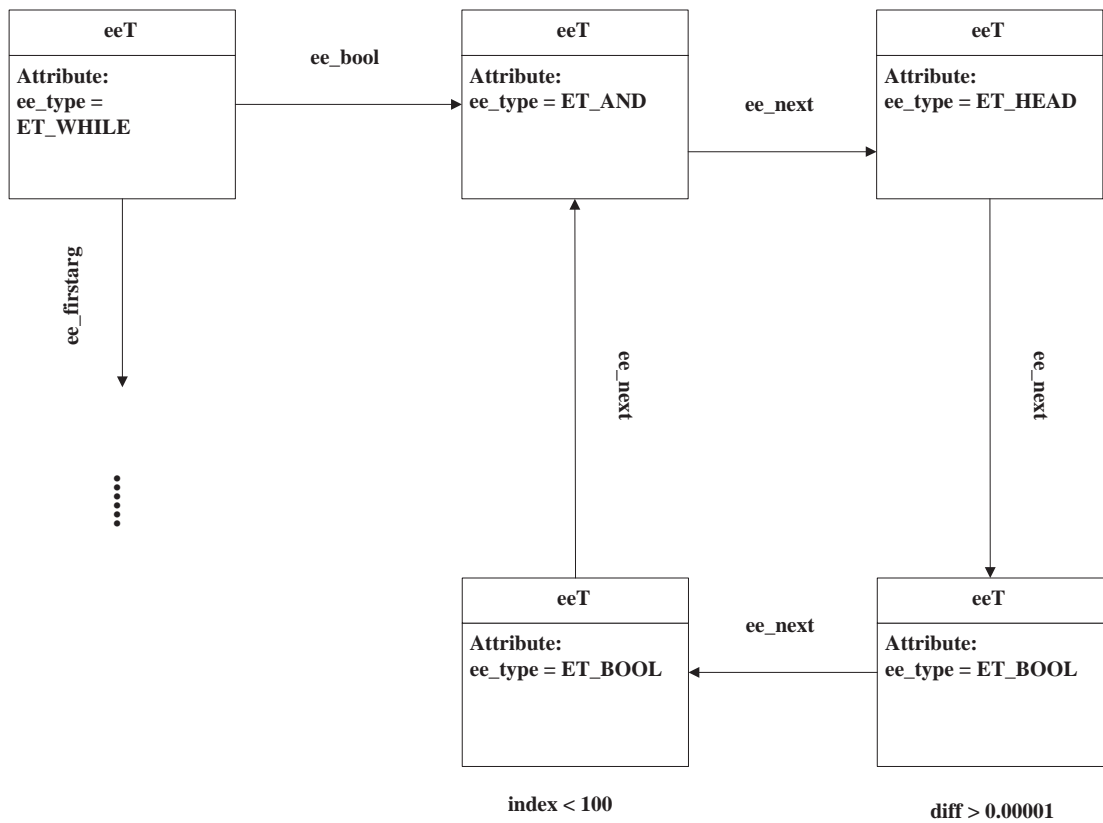


## Advanced Expression VI

### Extended Expression II ---- while- statement

```

while (diff > 0.00001 and index < 100)
<<while- statement >
  <loop >
  <if- statement >
  <bind simple_var expression >
  <expr expression { expression ... }>
  <expression >
  <epsilon e_type expression >>
end
  
```

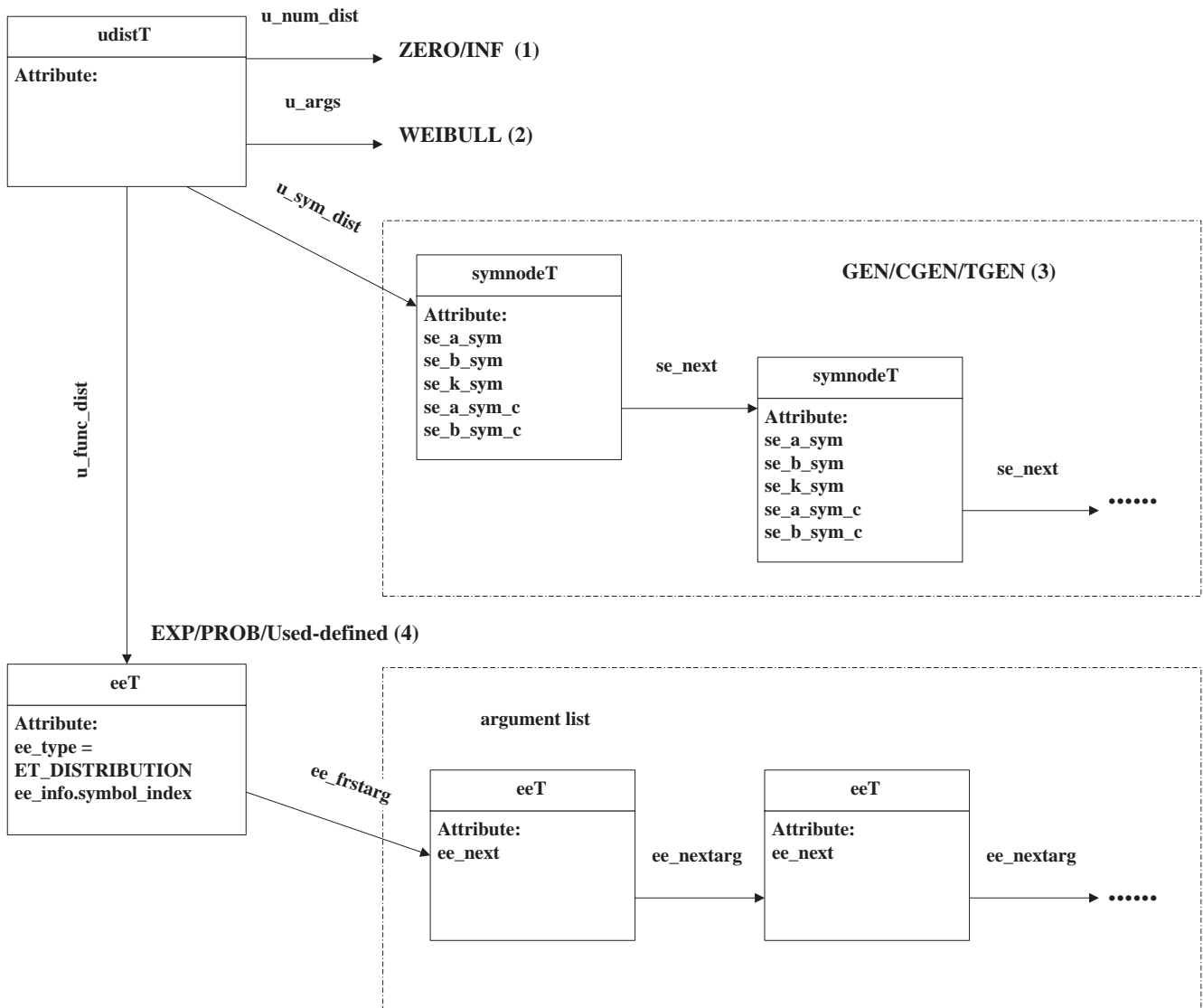




# Advanced Expression VII

## Distribution Expression:

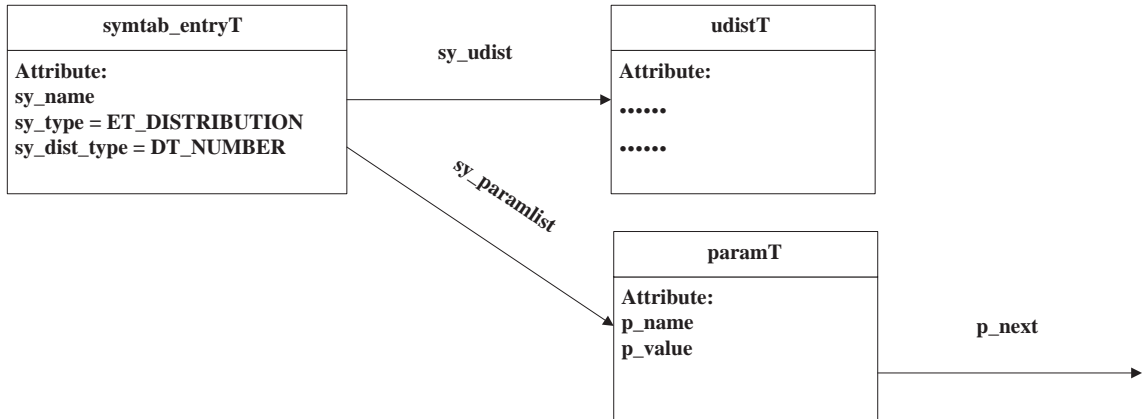
- ZERO/INF (1)
- WEIBULL (2)
- GEN/CGEN/TGEN (3)
- EXP/PROB/Used-defined (4)



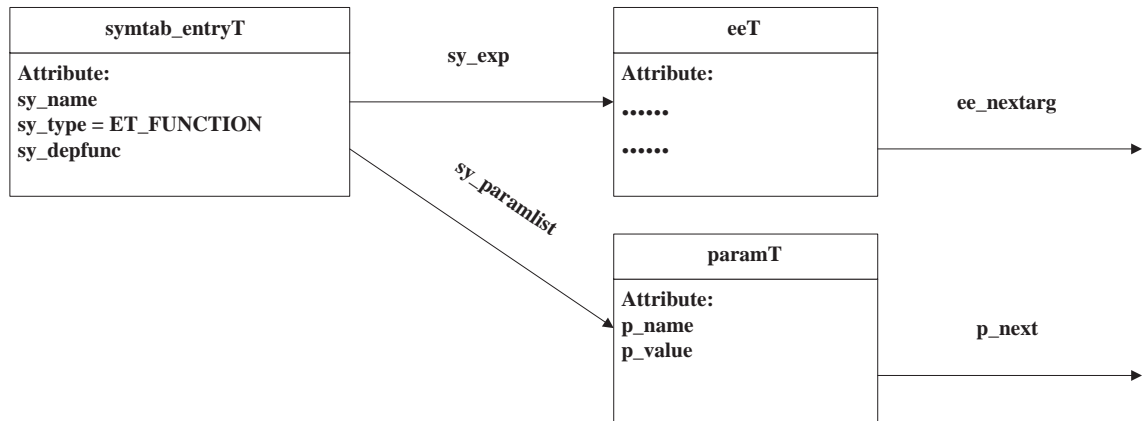
# Symbol Table

## symP symtab

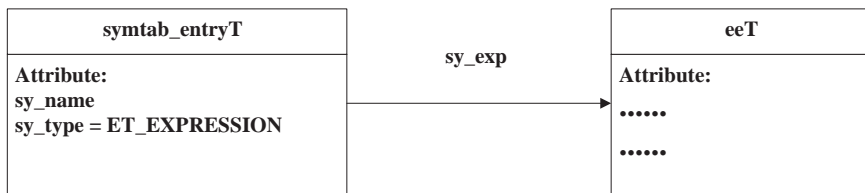
### Distribution Function:



### User Defined Function:

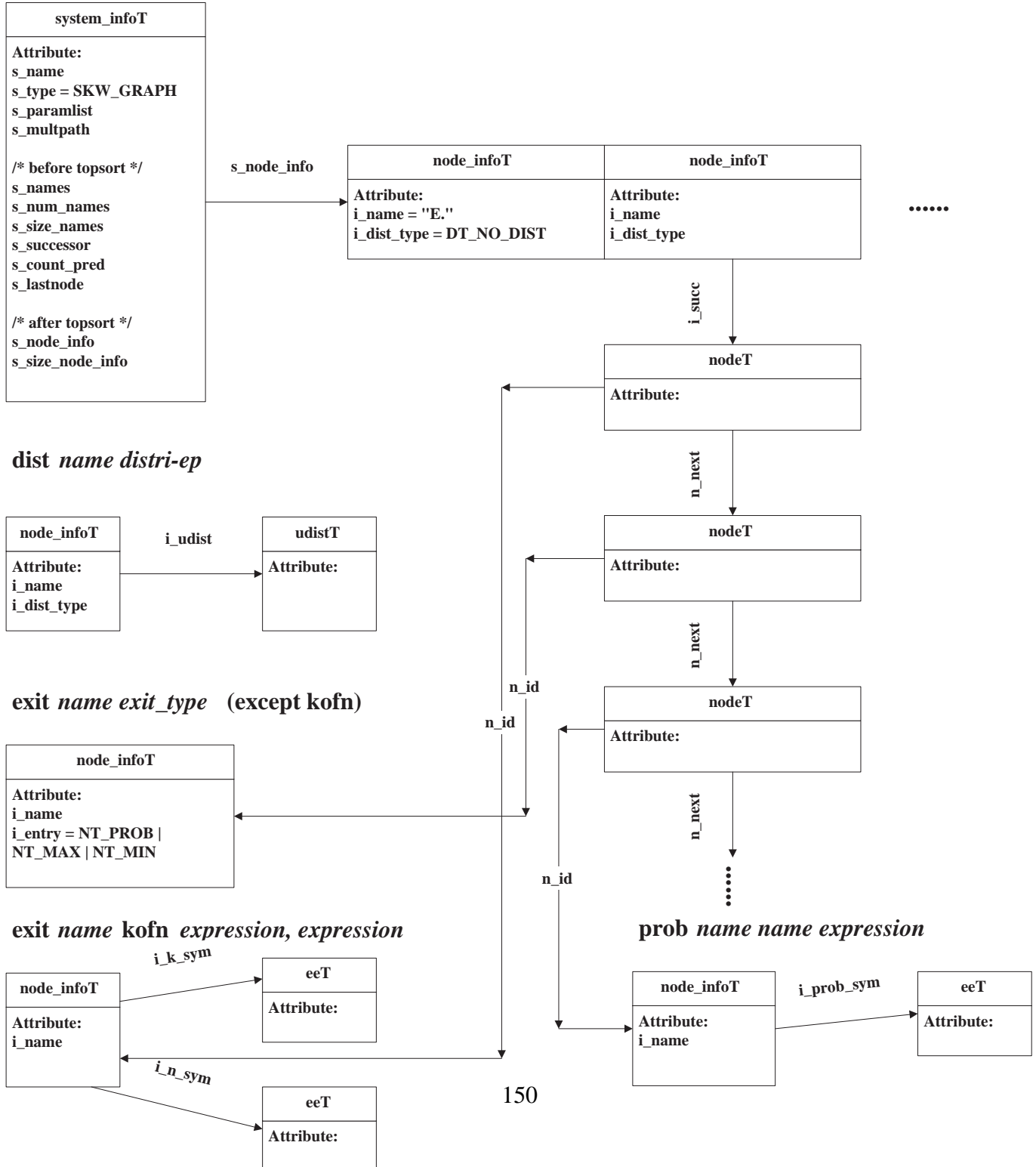


### var *defined\_var expression*



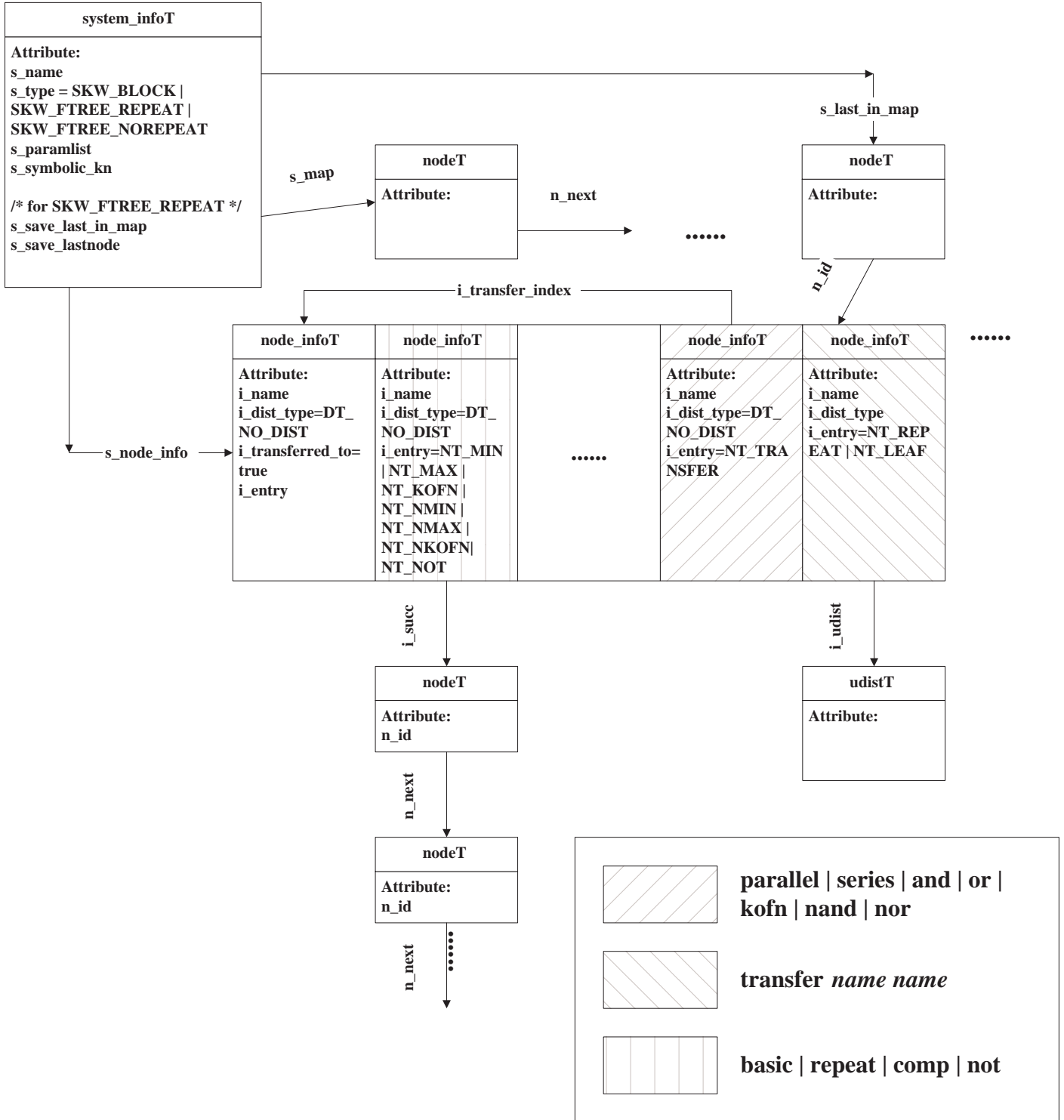
# System - Graph

## system\_infoP system\_info

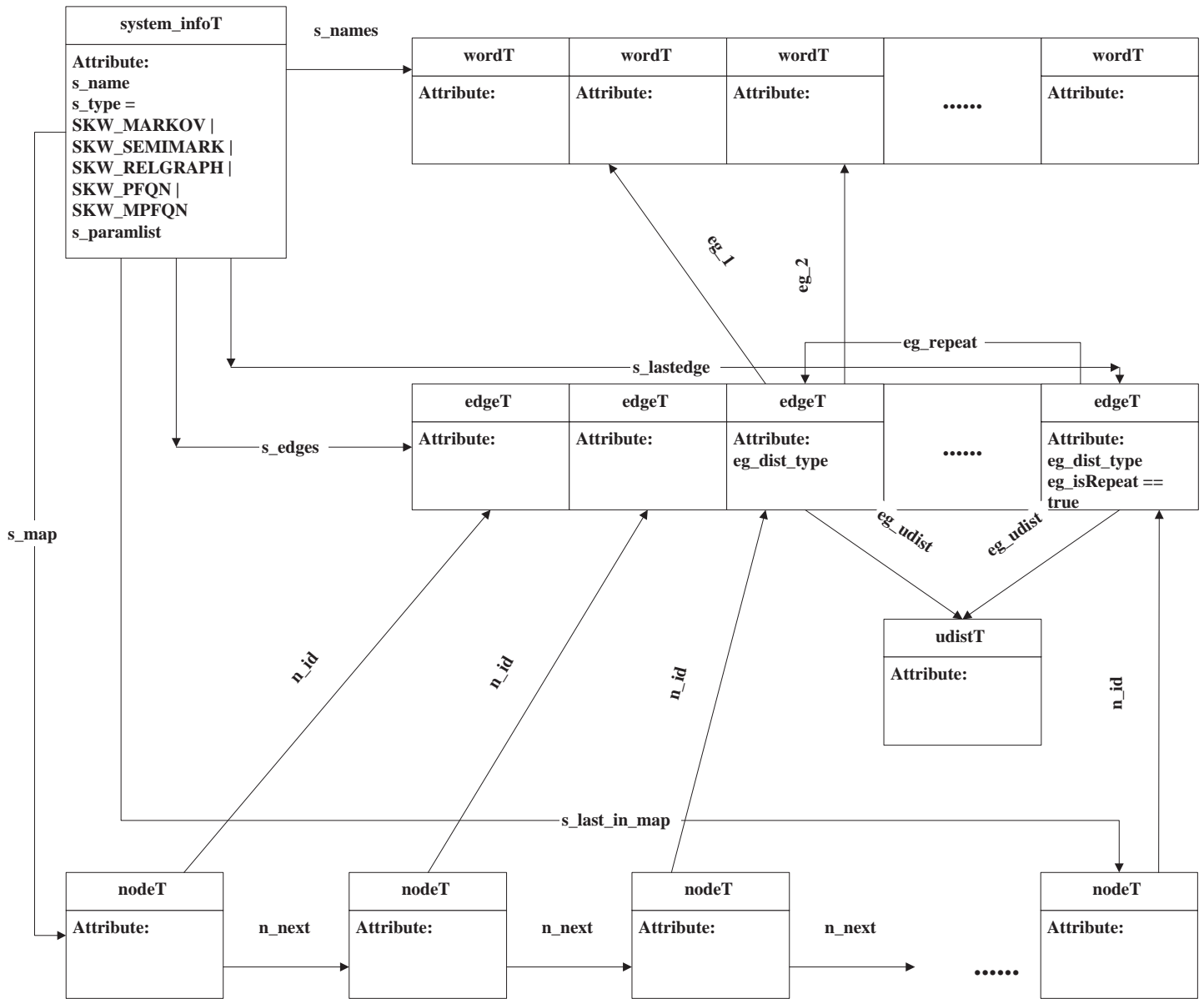


# System - Block | Fault Tree | MFT

## system\_infoP system\_info

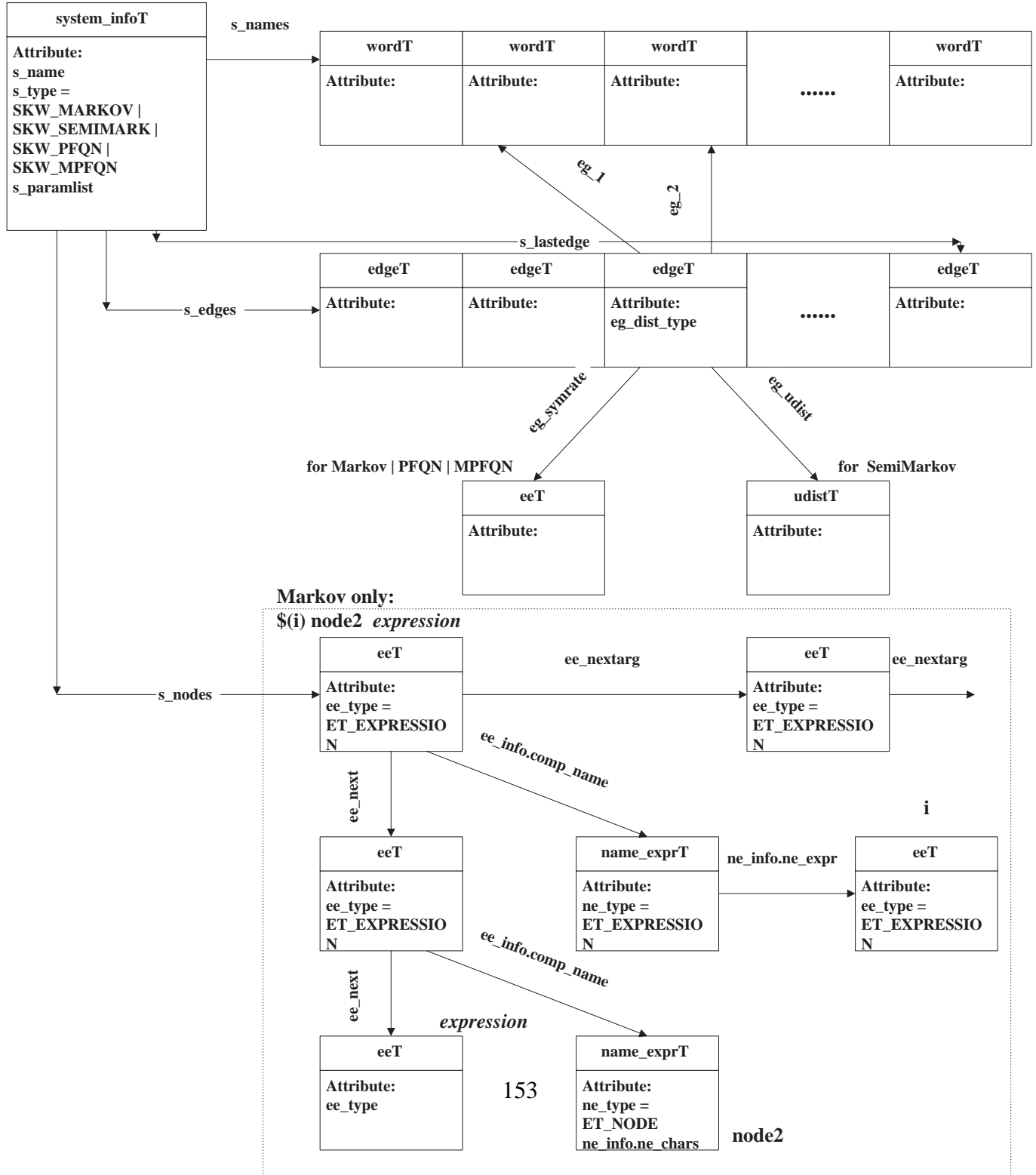


## System - Reliability Graphs



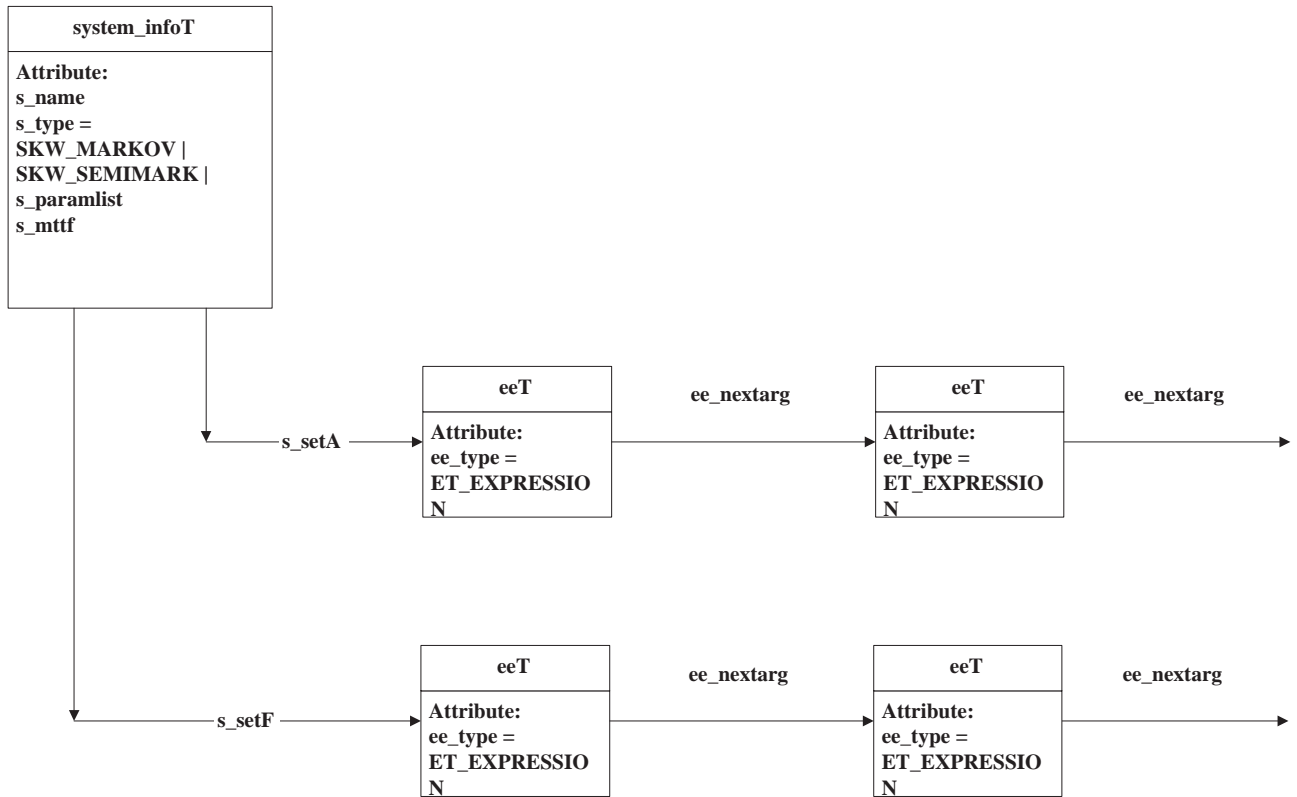
# System - Markov Chain\* | Semi Markov Chain | PFQN\* | MPFQN\*

\* means basic or partial data structure



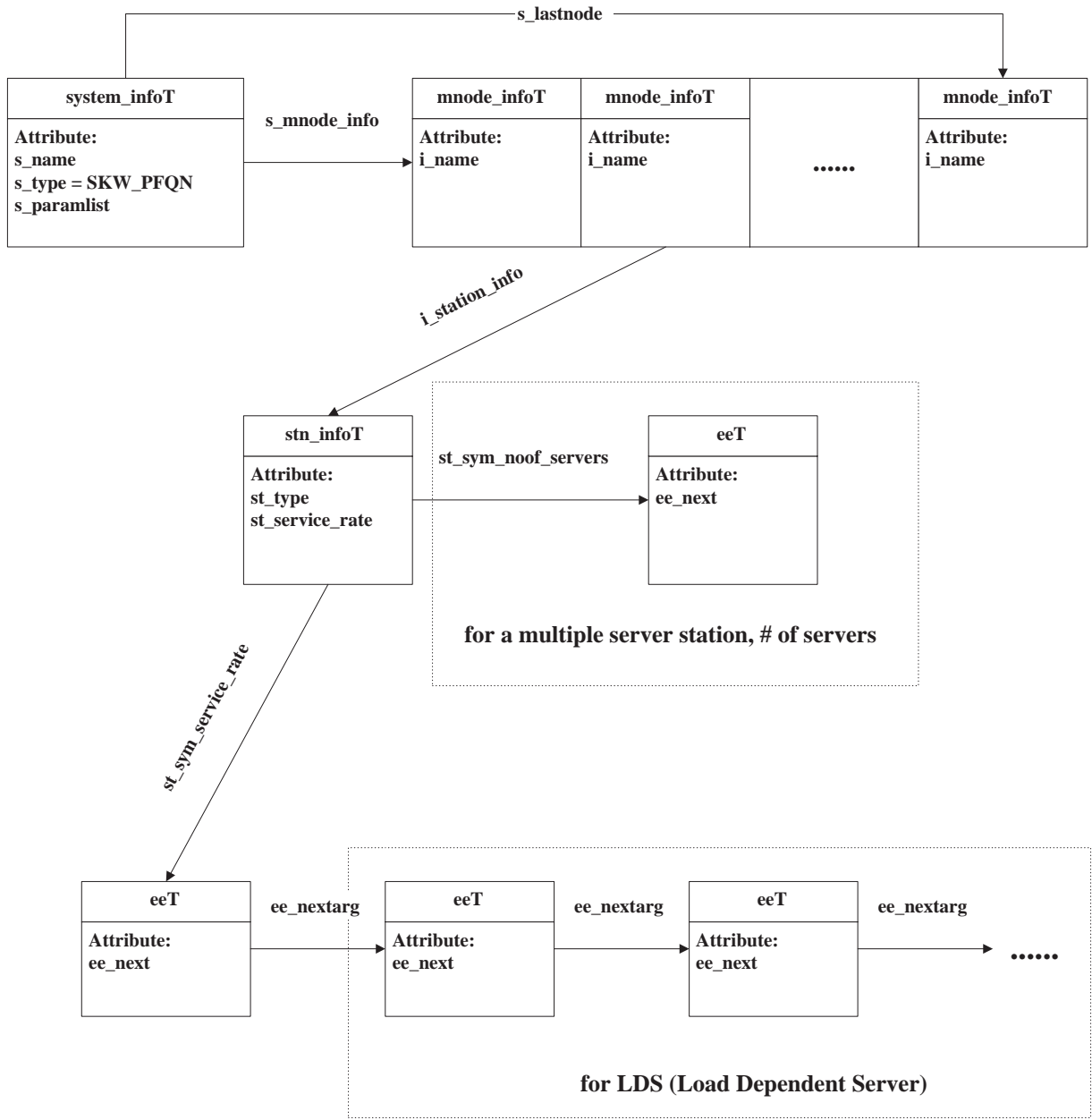


## System - Fast MTTF in Markov Chain| Semi-Markov

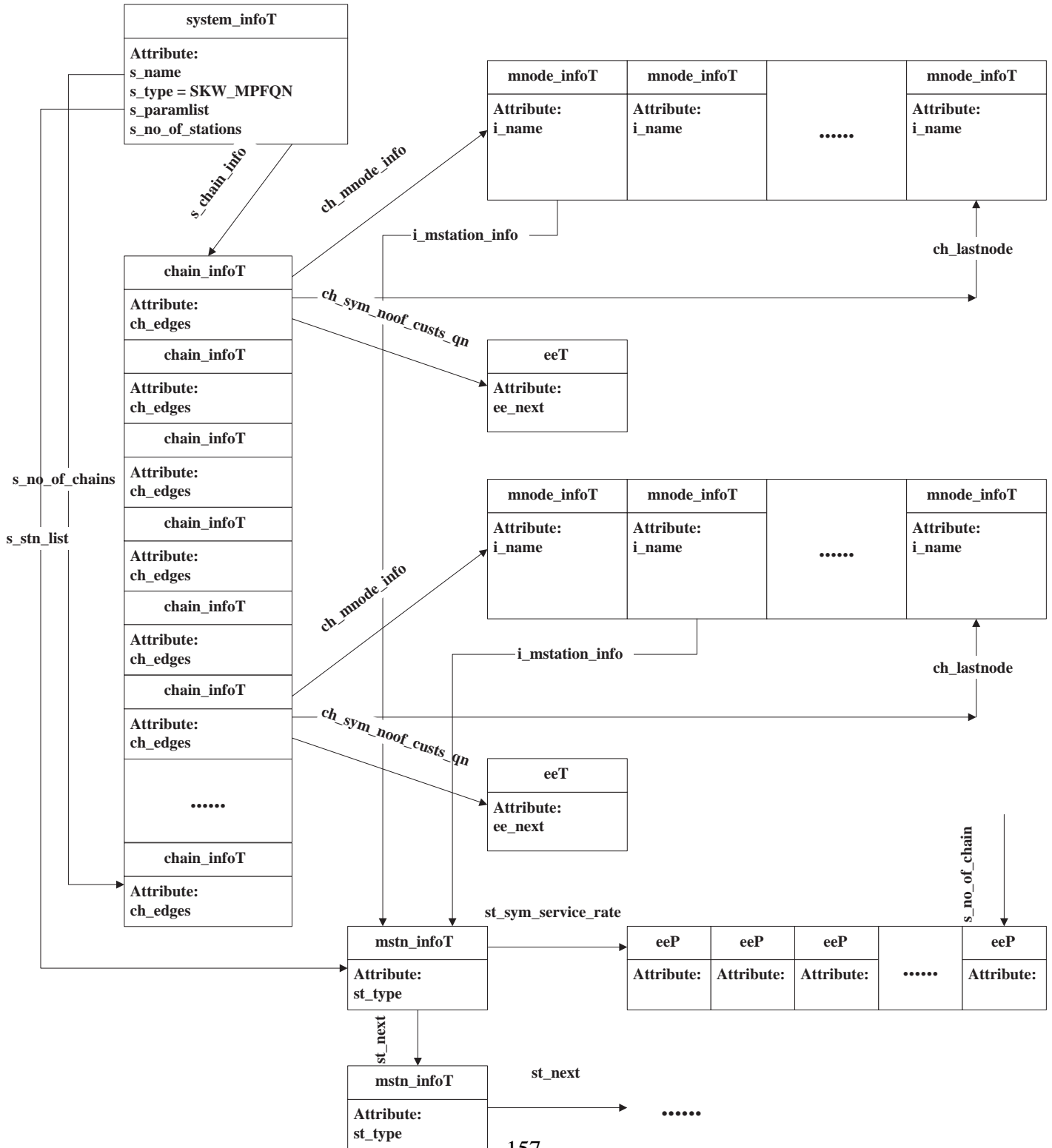




## System - PFQN after mtopsort()

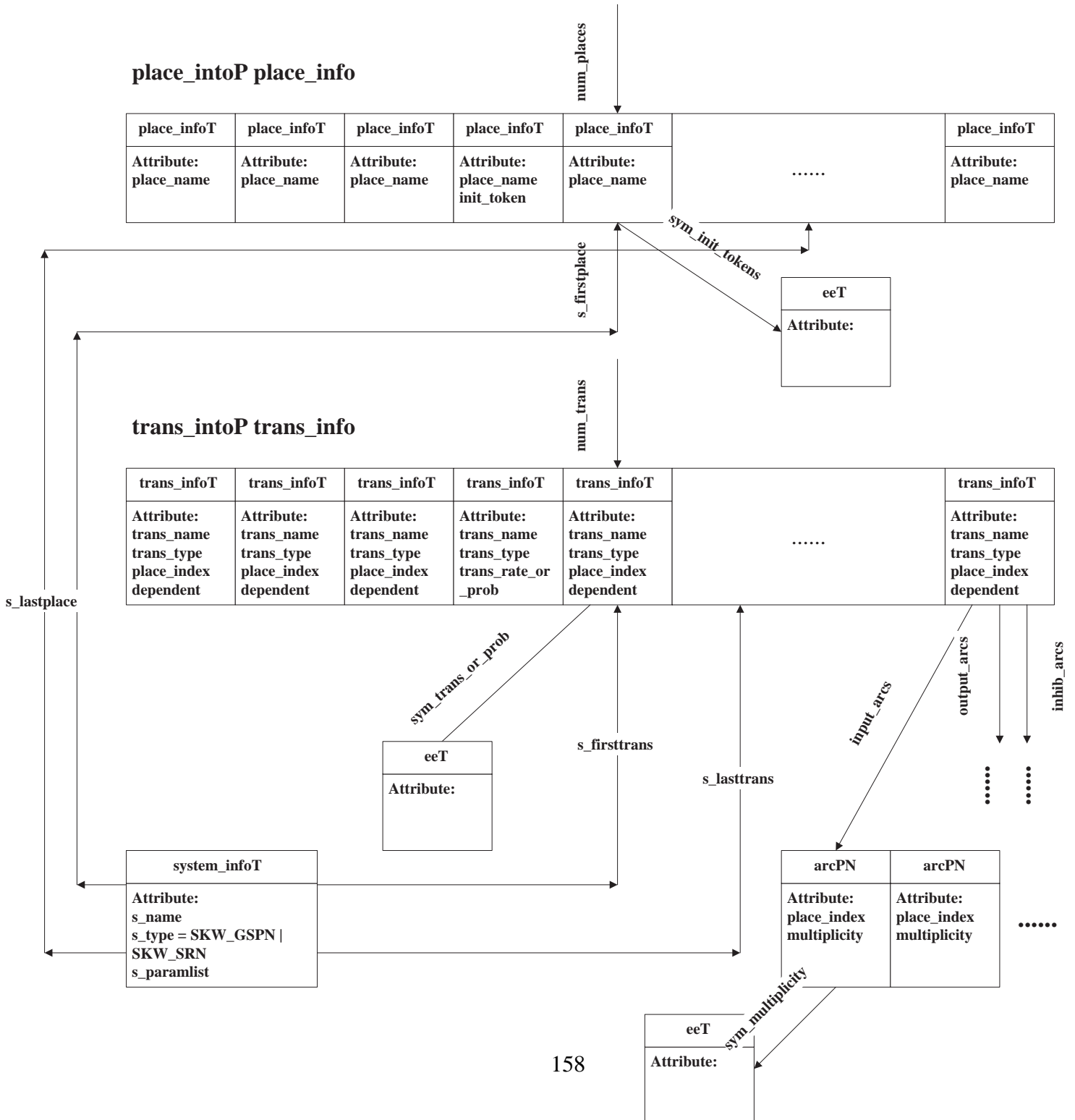


## System - MPFQN after mtopsort()

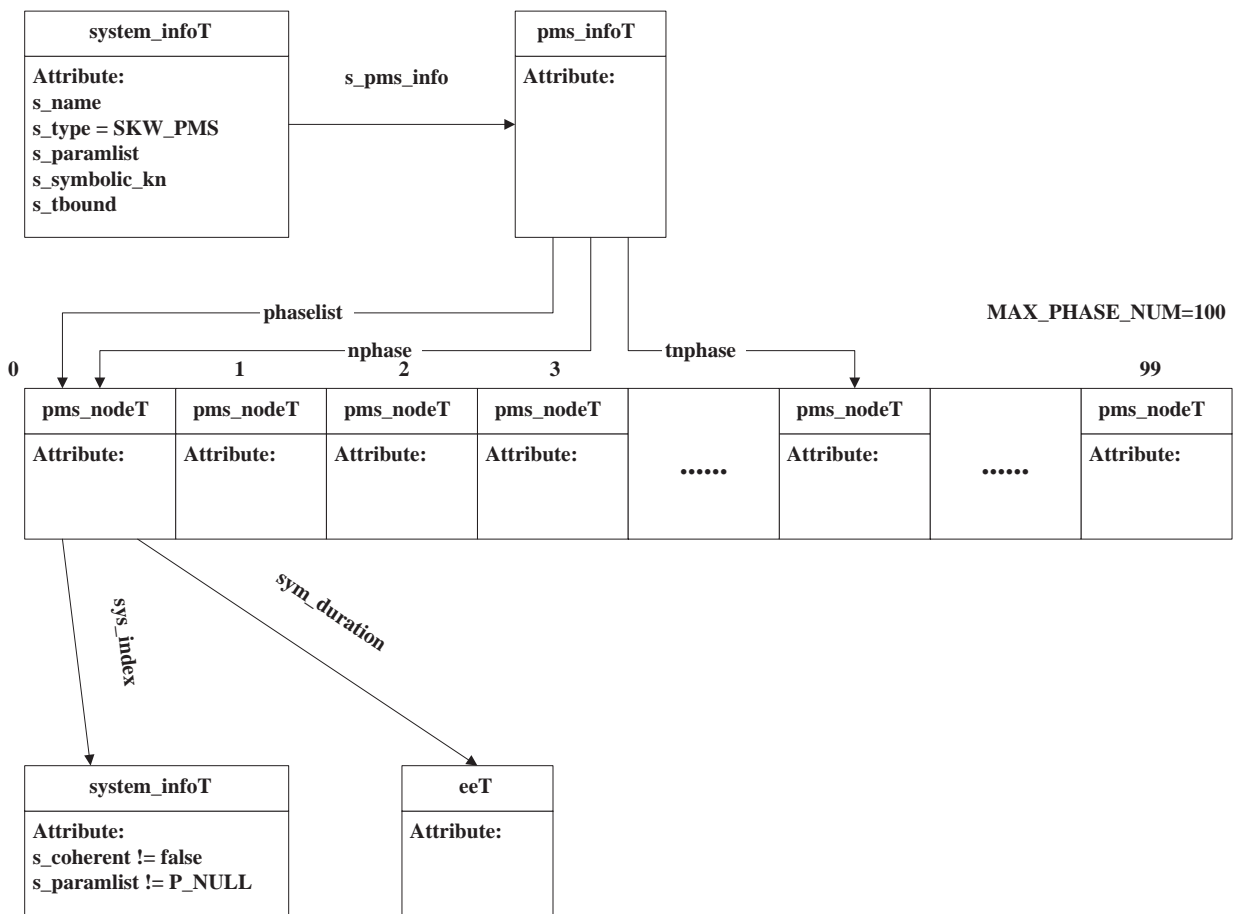


# System - GSPN | SRN

## system\_infoP system\_info



## System - PMS



## **Appendix B**

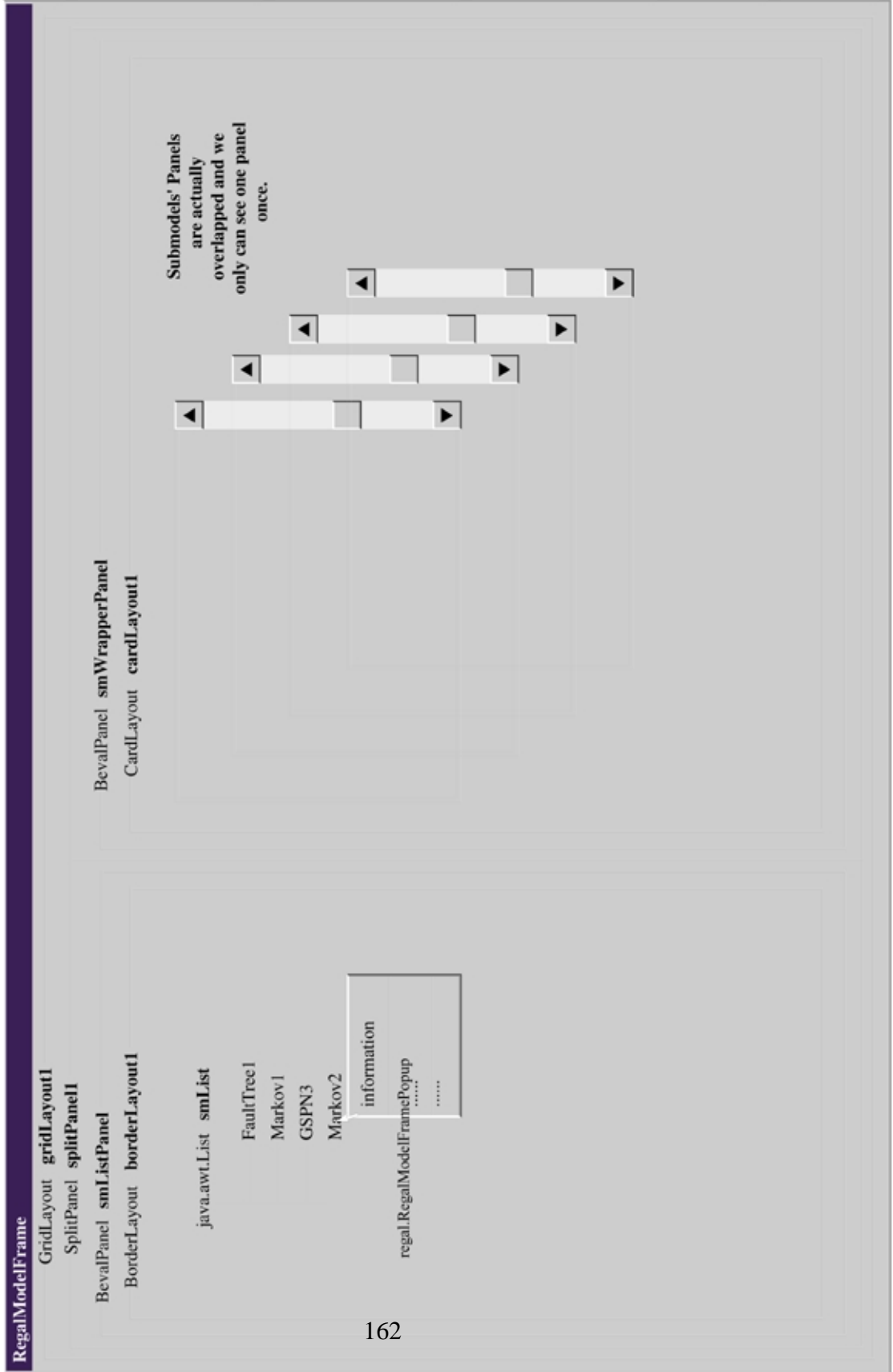
### **SHARPE GUI Documentation**

This appendix is a partial SHARPE GUI document. The first page is the object model [13] of the GUI program. The second is the window layout of the main window. The third is the object model of the analysis window. The last is the window layout of the analysis window.



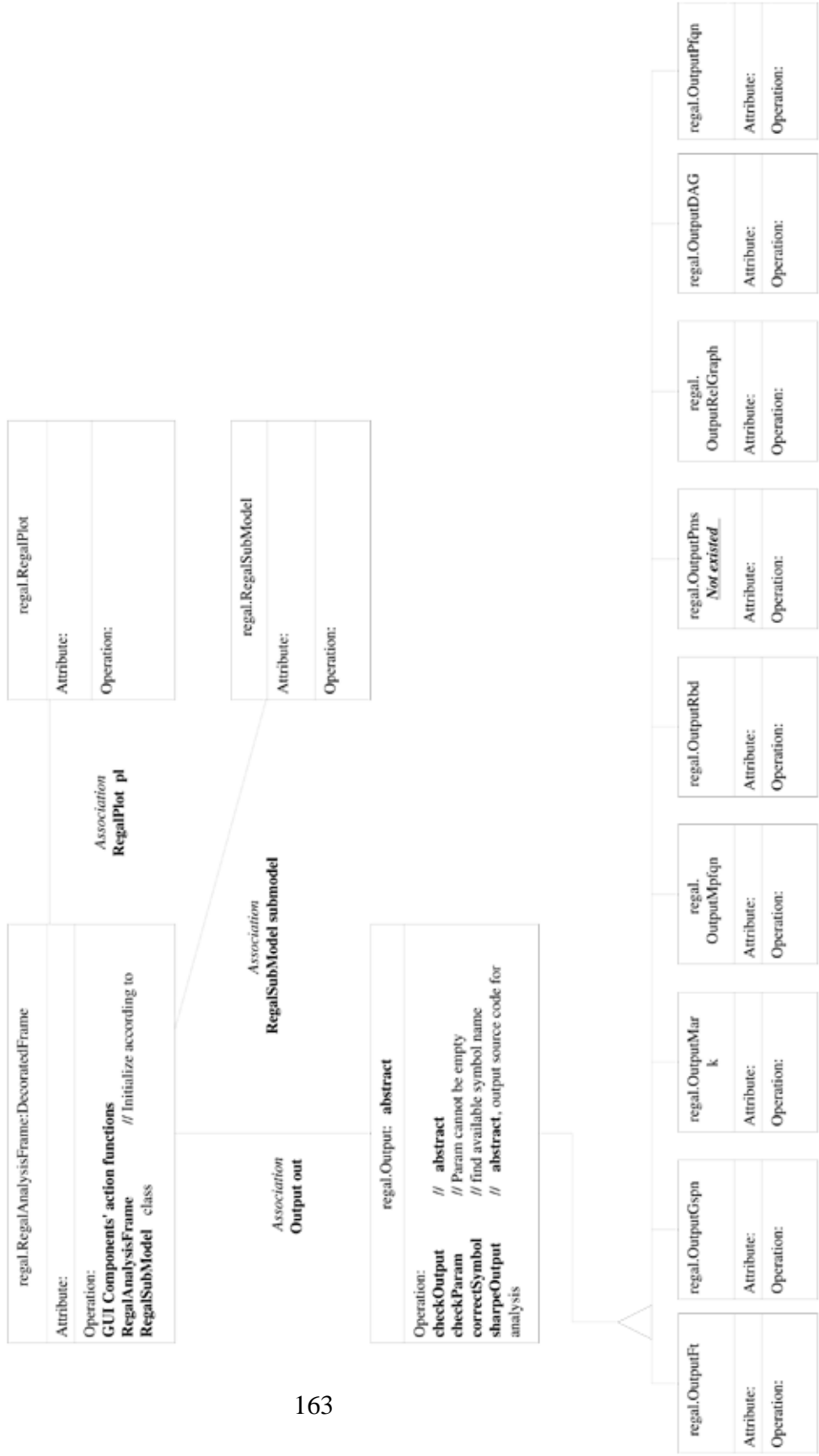
# SHARPE GUI Class Architecture II

## GUI Components' Layout in regal.RegalModelFrame



# SHARPE GUI Class Architecture III

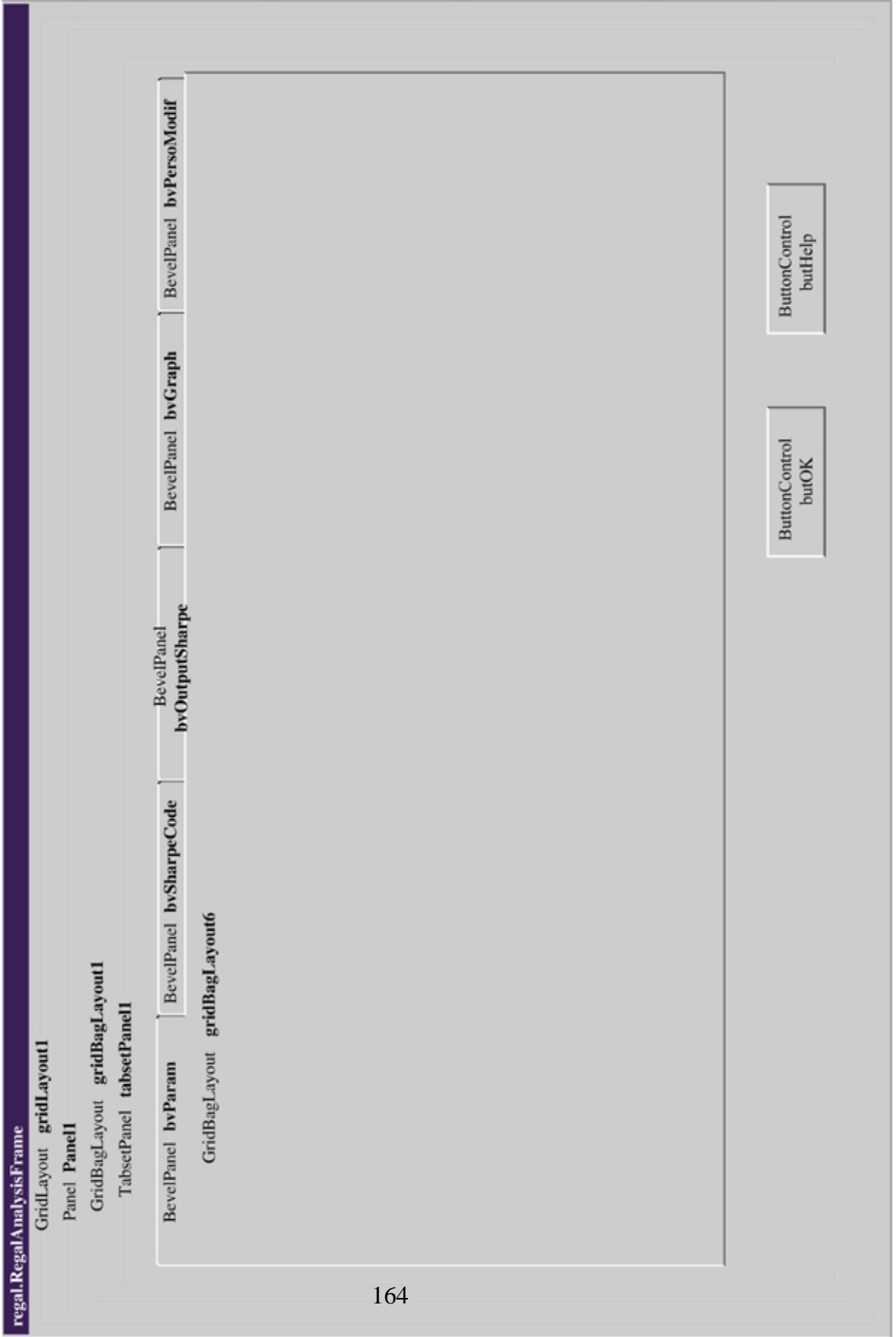
## Class `regal.RegalAnalysisFrame`





# SHARPE GUI Class Architecture IV

## GUI Components' Layout in regal.RegalAnalysisFrame



# Appendix C

## SHARPE Examples

Here, more SHARPE examples are listed.

### C.1 Fault Tree Examples

#### C.1.1 SHARPE File — *ftree\_n/example12*

\*Example 12

\*Author Luo Tong

\*To test the MVI in fault tree with inverse gates

\* TEST\_KEY sysunrel: 3.0000e-01

\* version using only repeated components

ftree ft

repeat a prob(0.3)

repeat b prob(0.4)

basic c prob(0.8)

and d a b

nand f a d

or e d b

or g f e

and h a g

nor i g c

or z h i

end

var sysunrel pzero(ft)

expr sysunrel

end

### **C.1.2 SHARPE File** — *ftree\_n/xnkofn1*

ftree kn1

repeat r exp(3.2)

basic a exp(7)

basic b exp(4)

basic c exp(5)

basic d exp(11)

kofn abcd 2,4, a b c d

not nabcd abcd

and top nabcd r

end

ftree kn2

repeat r exp(3.2)

basic a exp(7)

basic b exp(4)

basic c exp(5)

basic d exp(11)

nkofn abcd 2,4, a b c d

and top abcd r

end

cdf(kn1)

cdf(kn2)

end

### C.1.3 SHARPE File — *ftreebdd1/mincut*

ftree dsp70

basic a prob(q)

basic b prob(q)

basic c prob(q)

basic d prob(q1)

or t3 a b

and t1 t3 d

transfer d1 d

and t2 c d1

or t0 t1 t2

end

bind

q 0.25

q1 0.30

end

mincuts(dsp70)

expr sysprob(dsp70)

ftree f\_long

basic a 0123456789012345678901234567890123456789 exp(3.1)

basic b exp(4.2)

basic c exp(3.7)

basic d exp(7.2)

basic e exp(2)

basic f exp(1.2)

basic g exp(0.8)

basic x exp(3)

basic y exp(4)

```
transfer e1 e
or BE b e
and A a0123456789012345678901234567890123456789 BE
kofn K1 1,3, a0123456789012345678901234567890123456789 x y
kofn K2 2,4, g c d a0123456789012345678901234567890123456789
or EG e1 g
or FC f c
and E EG FC
or top A K1 K2 E
end
```

```
mincuts(f_long)
```

```
expr mean(f_long)
```

```
ftree f_repeat (k1,k2)
```

```
basic a exp(3.1)
```

```
basic b exp(4.2)
```

```
basic c exp(3.7)
```

```
basic d exp(7.2)
```

```
basic e exp(2)
```

```
basic f exp(1.2)
```

```
basic g exp(0.8)
```

```
basic x exp(3)
```

```
basic y exp(4)
```

```
transfer e1 e
```

```
or BE b e
```

```
and A a BE
```

```
kofn K1 k1,3, a x y
```

```
kofn K2 k2,4, g c d a
```

```
or EG e1 g
```

```
or FC f c
```

```
and E EG FC
or top A K1 K2 E
end
```

```
mincuts(f_repeat;1,2)
```

```
expr mean(f_repeat;1,2)
end
```

### **C.1.4 SHARPE File — *ftree\_bdd2/impt***

```
verbose on
```

```
ftree tree0(x)
repeat c1 exp(0.1)
basic c2 exp(0.2)
basic c3 exp(x)
basic c4 exp(0.1)
and and1 c1 c2
and and2 c3 c4
or top and1 and2
end
```

```
bdd off
cdf(tree0;0.3)
```

```
bdd on
expr bimpt(2; tree0, c1;0.3)
expr bimpt(2; tree0, c1;0.2)
expr bimpt(2; tree0, c1;0.1)
expr bimpt(2; tree0, c1;0.3)
expr cimpt(2; tree0, c1;0.3)
```

```
expr simpt(tree0, c1;0.3)
```

```
cdf(tree0;0.3)
```

```
end
```

## **C.2 Examples of Reliability Graphs**

### **C.2.1 SHARPE File — *relgraphbdd2/mincuts***

```
relgraph bridge
```

```
1 2 exp(1)
```

```
1 3 exp(2)
```

```
2 3 exp(3)
```

```
3 2 exp(2.3)
```

```
2 4 exp(4.7)
```

```
3 4 exp(5)
```

```
end
```

```
mincuts(bridge)
```

```
end
```

### **C.2.2 SHARPE File — *relgraph/minpath***

```
bdd off
```

```
relgraph bridge0
```

```
1 2 prob(q)
```

```
1 3 prob(q)
```

```
2 3 prob(q)
```

```
3 2 prob(q)
2 4 prob(q)
3 4 prob(q)
end
```

```
bind
q 0.1
end
```

```
minpaths(bridge0)
```

```
expr 1 - sysprob(bridge0)
```

```
end
```

### **C.2.3 SHARPE File — *relgraphbdd2/reltest1***

```
format 8
```

```
relgraph bridge0
```

```
1 2 prob(q1)
2 4 prob(q2)
1 3 prob(q1)
3 4 prob(q2)
bidirect
2 3 prob(q3)
end
```

```
bind
q1 0.01
q2 0.015
q3 0.02
```



```

end

expr sysprob(bridge0)
expr simpt(bridge0, 3, 4)
expr simpt(bridge0, 1, 2)
expr simpt(bridge0, 2, 4)
expr simpt(bridge0, 2, 3)
expr simpt(bridge0, 3, 2)
expr bimpt(10; bridge0, 3, 4)
expr cimp(10; bridge0, 3, 4)

end

```

## C.3 Examples of Fast MTTF [6]

### C.3.1 SHARPE File (Markov Chain) — *fastmttf/m6*

```

format 8
bind lambda 0.1
bind mu 1

markov t2 readprobs
6_0 5_1 6*lambda

5_1 5_0 1*lambda
5_1 4_2 5*lambda

5_0 4_1 5*lambda
5_0 6_0 mu

4_2 3_3 4*lambda
4_2 4_1 2*lambda

```

4.1 3.2 4\*lambda

4.1 4.0 1\*lambda

4.1 5.1 mu

4.0 3.1 4\*lambda

4.0 5.0 mu

3.3 2.4 3\*lambda

3.3 3.2 3\*lambda

3.2 2.3 3\*lambda

3.2 3.1 2\*lambda

3.2 4.2 mu

3.1 2.2 3\*lambda

3.1 3.0 1\*lambda

3.1 4.1 mu

3.0 2.1 3\*lambda

3.0 4.0 mu

2.4 1.5 2\*lambda

2.4 2.3 4\*lambda

2.3 1.4 2\*lambda

2.3 2.2 3\*lambda

2.3 3.3 mu

2.2 1.3 2\*lambda

2.2 2.1 2\*lambda

2.2 3.2 mu

2.1 1\_2 2\*lambda

2.1 2\_0 1\*lambda

2.1 3\_1 mu

2.0 1\_1 2\*lambda

2.0 3\_0 mu

1.5 0\_6 1\*lambda

1.5 1\_4 5\*lambda

1.4 0\_5 1\*lambda

1.4 1\_3 4\*lambda

1.4 2\_4 mu

1.3 0\_4 1\*lambda

1.3 1\_2 3\*lambda

1.3 2\_3 mu

1.2 0\_3 1\*lambda

1.2 1\_1 2\*lambda

1.2 2\_2 mu

1.1 0\_2 1\*lambda

1.1 1\_0 1\*lambda

1.1 2\_1 mu

1.0 0\_1 1\*lambda

1.0 2\_0 mu

0.6 0\_5 6\*lambda

0.5 0\_4 5\*lambda

0.5 1\_5 mu

0.4 0\_3 4\*lambda

0.4 1\_4 mu

0.3 0\_2 3\*lambda

0.3 1\_3 mu

0.2 0\_1 2\*lambda

0.2 1\_2 mu

0.1 1\_1 mu

0.1 0\_0 1\*lambda

0.0 1\_0 mu

end

6.0 1

end

fastmttf

6.0 READA

2.4 READA

\*3\_3 READA

0.0 READF

end

expr fastmttf(t2)

end

### **C.3.2 SHARPE File (Semi-Markov Chain) — *fastmttf/semif***

semimark abc2

```
m1 m2 exp(1.2)
m2 m3 exp(0.8)
m1 m3 exp(1.4)
m2 m1 exp(0.3)
m3 m1 exp(1.5)
m3 m4 exp(2.5)
m4 m1 exp(1.0)

end

m1 1
end

fastmttf
m1 READA
m2 READA
m3 READF
end

expr fastmttf(abc2)
end
```

## C.4 SRN Example

### C.4.1 SHARPE File — *srn/mtta*

\* Translate from sensi.c of SPNP6

```
format 8

bind
thinktime 1
CPUrate 0.01
```

```

rate1    0.04
rate2    0.05
TK       2
exit_prob 0.01
out1_prob 0.30
out2_prob 0.69
lambda   1.0/CPUrate
theta    1.0
end

srn mttatest()
* Places
think   0
CPU     TK
decide  0
use1    0
use2    0
end
* Timed transitions
go placedep think 1.0/thinktime
CPUdone ind lambda
done1 ind 1.0/rate1*theta
done2 ind 1.0/rate2*theta
end
* Immediate transitions
exit1 ind exit_prob
out1 ind out1_prob
out2 ind out2_prob
end
* Input arcs
think go 1
CPU CPUdone 1

```

```

decide  exit1  1
decide  out1  1
decide  out2  1
use1    done1 1
use2    done2 1
end

* Output arcs
go  CPU  1
CPUdone  decide  1
exit1  think  1
out1   use1  1
out2   use2  1
done1  CPU  1
done2  CPU  1
end

* Inhibitor arcs
think  go  TK
end

func  refunc()
if (#(think) == TK)
0
else
1
end
end

expr  sm_cexrinf(mttatest; refunc)
expr  mta(mttatest)

end

```

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